Decision Making and Numeracy: The Role of Context in Adaptive Strategy Selection

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Abstract

In the real world, making good decisions under risk demands flexibility in reactions to dynamically changing task demands. Literature indicates that skilled decision makers have better knowledge of the context in which they operate, and more importantly, compared to less skilled decision makers, they take into account the associated environmental and cognitive limitations to make adaptive decisions. Here, the term "context" signifies the constraints (both cognitive and environmental) associated with the task structure presented to the participants. On the other hand, a group of decisions can be considered "adaptive" when, as a collective, it outperforms the existing normative prediction or matches it without utilizing a similar amount of resources. Put simply, skilled decision makers better understand the statistical regularities in the decision environment and use their superior decision repertoire to make better choices.

In this thesis, I extended this finding to the field of numeracy and adaptive decision making using six empirical studies and one simulation study documented over three published articles. In all empirical studies, participants were presented with choice problems (using decision from description and decision from experience paradigms) alongside a scale that measured their numeric ability (Berlin Numeracy Test). The study procedure was similar across the whole thesis, where the aim of the study, payment terms, and research protocols were explicitly instructed to the participants. The experimental procedures of all studies reported in the thesis were approved by the Ethical Review Board at SWPS University.

In the first published article, I replicated the finding of Traczyk, Sobkow, et al. (2018) where the authors argued that more numerate individuals, compared to less numerate individuals, better adaptively modulate their decision strategy following the changes in relative difference in payoff structure (High-payoff vs. Low-payoff condition). This was one of the very first reported findings showing that individuals with better numeracy skills tend to make more adaptive decisions. Therefore, it was of great interest to the field of numeracy and adaptive decision making to see if this result can be replicated or not. In the first study, I demonstrated that the reported moderation effect of numeracy on the relationship between payoff conditions and adaptive decision making is robust and replicable. The result demonstrated that individuals with high statistical numeracy tended to use more energy-intensive decision-making strategies when faced with important choices (i.e., High-payoff condition) compared to individuals with low statistical numeracy. However, when the decision was less important (i.e., Low-payoff condition), the more numerate individuals tended to use quicker, less effortful heuristic strategies to save time and mental effort.

Although I successfully replicated the findings from Traczyk, Sobkow, et al. (2018), I identified a few limitations (i.e., lack of trade-off between decision strategies, asymmetry in the distribution of absolute difference in the expected value, and varied difficulty level between conditions) that could affect the overall conclusion. Therefore, in the second publication consisted of two empirical studies, I planned to mitigate the limitation and tested the effect of adaptive modulation in decision strategy in a more

controlled environment. I found that the payoff conditions alone did not invoke adaptive strategy selection, irrespective of numeracy levels. More numerate individuals were better at maximizing expected reward following the frame of reference of absolute difference (as captured by the absolute expected value difference distribution) and not the relative difference in reward (as captured by both payoff conditions). In the second study, I successfully dissociated the relationship between payoff conditions and adaptive strategy selection.

In the last publication, instead of providing contextual information in the form of numerical difference (i.e., a significant contrast in relative difference in reward between two payoff conditions), I used a more straightforward environmental limitation (i.e., time constraints) where making adaptive decisions is simple yet contrary to existing normative principles of decision-making literature. In other words, I created a task environment where making recurring suboptimal choices is adaptively rational. I empirically tested this specific pattern of behavior in three empirical studies. I found that individuals with higher numeracy, compared to individuals with lower numeracy, adaptively modified their exploration strategies in response to the alterations in the task environment (constraints vs no constraints environment). Specifically, more numerate individuals engaged in a more dynamic and flexible information sampling and employed smart search strategies, predominantly focusing on reducing uncertainty and uncovering unobserved outcomes. This adaptability facilitated the development of a metacognitive understanding of both the structure of the task and the choice environment, leading to the predominant use of random choices to accumulate higher total rewards.

Overall, results from the three publications confirmed that the adaptive qualities of highly skilled decision makers could be generalized to those who are highly proficient with numbers (i.e., more numerate individuals) when dealing with numeric problems. Notably, these findings underscored the role context played in enabling more skilled decision makers to make superior decisions. Due to their superior numeric ability, more numerate individuals understood the resource limitations of the environment and modulated their decision strategy where they knowingly violated normative principles to outperform normative predictions. Therefore, interventions (i.e., nudging or boosting) created to aid decision-making should recognize that, under certain conditions, habitual suboptimal choices can lead to improved decision-making.

Streszczenie

W codziennych sytuacjach podejmowanie dobrych decyzji w warunkach ryzyka wymaga elastyczności w reagowaniu na dynamicznie zmieniajace sie wymagania zadania. Literatura wskazuje, że doświadczeni decydenci (ang. skilled decision makers) maja lepsza wiedze na temat kontekstu, w którym działaja. Co ważniejsze, w porównaniu z mniej doświadczonymi decydentami, uwzgledniaja zwiazane z tym ograniczenia środowiskowe i poznawcze, aby podejmować adaptacyjne decyzje. Tutaj termin "kontekst" oznacza ograniczenia (zarówno poznawcze, jak i środowiskowe) zwiazane ze struktura zadania, w obliczu którego stoja decydenci. Z drugiej strony, decyzje można uznać za "adaptacyjne", gdy jako grupa wyborów przewyższaja przewidywania modelu normatywnego lub mu dorównuja, nie angażujac przy tym podobnej ilości zasobów poznawczych. Mówiac prościej, doświadczeni decydenci lepiej rozumieja statystyczne regularności w środowisku decyzyjnym i używaja bardziej zróżnicowanego repertuaru strategii decyzyjnych, aby dokonywać lepszych wyborów.

W niniejszej rozprawie doktorskiej rozszerzam to odkrycie na obszar zdolności numerycznych i adaptacyjnego podejmowania decyzji, prezentujac wyniki sześciu badań empirycznych i jednego badania symulacyjnego, które zostały opisane w trzech artykułach opublikowanych w recenzowanych czasopismach. We wszystkich badaniach empirycznych uczestnikom przedstawiano problemy decyzyjne (korzystajac z zadań opartych na paradygmatach decyzji na podstawie opisu" i decyzji na podstawie doświadczenia") wraz z narzedziem do pomiaru zdolności numerycznych (Berliński Test Zdolności Numerycznych). Procedura była zbliżona we wszystkich badaniach raportowanych w pracy. Cel badania, warunki wynagrodzenia i procedura eksperymentalna były zawsze jasno przedstawione uczestnikom w każdym z badań. Procedury wszystkich badań raportowanych w rozprawie zostały pozytywnie zaopiniowane przez Komisje Etyki Uniwersytetu SWPS.

W pierwszym opublikowanym artykule zreplikowałem wyniki badania Traczyk, Sobkow, et al. (2018), gdzie autorzy udowodnili, że osoby z wyższymi zdolnościami numerycznymi, w porównaniu z osobami z niższymi zdolnościami numerycznymi, adaptacyjnie dostosowuja swoja strategie decyzyjna w odpowiedzi na zmiany w wzglednej różnicy w strukturze wypłat (warunek wysoka wypłata vs niska wypłata). To jedno z pierwszych odkryć pokazujacych, że osoby z wyższymi zdolnościami numerycznymi maja tendencje do podejmowania bardziej adaptacyjnych decyzji, dlatego ważna jest replikacja tego wyniku. Wykazałem, że raportowany efekt moderacji zdolności numerycznych na zwiazek miedzy warunkami wypłat a adaptacyjnym podejmowaniem decyzji jest powtarzalny. Moje wyniki wskazuja, że osoby z wyższymi zdolnościami numerycznymi maja tendencje do używania bardziej wymagajacych strategii decyzyjnych, gdy staja przed ważnymi wyborami (tj. warunek wysoka wypłata), w porównaniu do osób z niższymi zdolnościami numerycznymi. Jednak gdy decyzja jest mniej ważna (tj. warunek niska wypłata), osoby z wyższymi zdolnościami numerycznymi maja tendencje do używania szybszych, mniej angażujacych strategii heurystycznych, aby oszczedzać czas i wysiłek poznawczy.

Mimo że udało mi sie powtórzyć odkrycia przedstawione przez Traczyk, Sobkow, et

al. (2018), zidentyfikowałem także kilka ograniczeń w obszarze zastosowanej wcześniej metody badawczej (tj. brak przetargu miedzy strategiami decyzyjnymi, asymetria w rozkładzie bezwzglednej różnicy w oczekiwanej wartości, zróżnicowany poziom trudności miedzy warunkami), które mogły wpłynać na końcowy wniosek, płynacy z tego badania. Dlatego też w drugiej publikacji składajacej sie z dwóch badań empirycznych, planowałem zaadresować te ograniczenia i przetestować efekt adaptacyjnej selekcji strategii decyzyjnych w bardziej kontrolowanych warunkach. Stwierdziłem, że same warunki wypłat nie wywołuja adaptacyjnej selekcji strategii, niezależnie od poziomu umiejetności numerycznych. Osoby z wyższymi zdolnościami numerycznymi lepiej maksymalizuja oczekiwana nagrode w oparciu o punkt odniesienia różnicy bezwzglednej (uchwyconej w rozkładzie bezwzglednej różnicy oczekiwanej wartości), a nie wzglednej różnicy w nagrodzie (uchwyconej w obu warunkach wypłat). W tej serii badań, udało mi sie rozróżnić i opisać zależności miedzy warunkami wypłat a adaptacyjna selekcja strategii decyzyjnych.

W ostatniej publikacji, zamiast dostarczania badanym informacji kontekstowych w formie różnicy numerycznej (tj. kontrastu we wzglednej różnicy w wynagrodzeniu miedzy dwoma warunkami wypłat), użyłem prostszego ograniczenia środowiskowego (tj. ograniczenia czasowego), gdzie podejmowanie adaptacyjnych decyzji jest łatwiejsze, ale sprzeczne z założeniami modeli normatywnych. Innymi słowy, stworzyłem warunki zadania, w których podejmowanie powtarzajacych sie suboptymalnych wyborów jest adaptacyjnie racjonalne. Empirycznie przetestowałem ten specyficzny wzorzec zachowań w trzech badaniach empirycznych. Stwierdziłem, że osoby o wyższych zdolnościach numerycznych, w porównaniu do osób o niższych zdolnościach numerycznych, adaptacyjnie modyfikowały swoje strategie eksploracyjne w odpowiedzi na zmiany w środowisku zadania (środowisko z ograniczeniami vs bez ograniczeń). Konkretnie, angażowały sie w bardziej dynamiczne i elastyczne próbkowanie informacji oraz stosowały inteligentne strategie poszukiwania informacji, skupiajac sie głównie na redukcji niepewności i odkrywaniu nieobserwowanych wyników. Ta adaptacyjność ułatwiała rozwój metapoznawczego zrozumienia zarówno struktury zadania, jak i środowiska wyboru, prowadzac do opierania sie na losowych decyzjach w celu uzyskania wyższej końcowej nagrody.

Podsumowujac, wyniki z trzech publikacji potwierdzaja, że umiejetność adaptacyjnego podejmowania decyzji przez doświadczonych decydentów można uogólnić na osoby o wysokich zdolnościach numerycznych (tj. bardziej biegłe w przetwarzaniu informacji numerycznych) podczas rozwiazywania problemów o charakterze probabilistycznym. W szczególności, odkrycia te podkreślaja role kontekstu w umożliwianiu doświadczonym decydentom podejmowania lepszych decyzji. Osoby z wyższymi zdolnościami numerycznymi rozumieja ograniczenia zasobów środowiskowych i dostosowuja swoja strategie decyzyjna do wymagań problemu decyzyjnego, świadomie naruszajac założenia modeli normatywnych w celu uzyskania lepszych wyników. W zwiazku z tym interwencje behawioralne stworzone w celu wspomagania podejmowania decyzji powinny uwzgledniać, że w pewnych warunkach powtarzalne suboptymalne wybory moga prowadzić do podejmowania decyzji lepszych decyzji.

Contents

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Introduction

"Why this painting is here, my child can also make this"
-Overheard in a modern art museum

When I visit modern art museums, I often hear someone saying that they or their child could paint something similar to some displayed artworks. This reaction is particularly common when the installation follows a Minimalist philosophy (see Fig. 2.1).

Figure 2.1: A example of Minimalist painting. Ad Reinhardt, Abstract Painting no. 4, 1961, oil on linen, 60 1/8 x 60 1/4 in. (152.6 x 152.9 cm.), Smithsonian American Art Museum, Gift of S.C. Johnson & Son, Inc., 1969.47.71.

I believe this response stems from a lack of historical knowledge regarding the shifts in artistic trends and cultural evolution. Minimalist painting emerged in the late 1950s and early 1960s as a reaction against Abstract Expressionism, characterized by its emphasis on emotional intensity, gestural brushstrokes, and spontaneity. In contrast, Minimalist artists aimed to reduce their work to its most essential elements, focusing on fundamental aspects like color, form, line, and surface. They stripped away any extraneous details, aiming for purity and precision. Viewers who have this knowledge are better equipped to understand the intent and motive behind such minimalist paintings. Art pieces, especially modern art pieces, cannot be viewed in isolation; all art is a response, and unless the viewer is aware of the context, every modern art will seem incoherent, random, or worse, pretentious.

This argument can also be extended to the study of human decision making. Every decision is a response to a specific context. Without understanding the context in which individuals make decisions, researchers cannot accurately assess the quality of those decisions. For instance, consider the well-known phenomenon of *probability matching*. In a typical T-maze experiment, a single mouse starts at a point where food is randomly placed 80% of the time on the left side and 20% on the right side. The optimal strategy for the mouse would be to always turn left. However, observations show that the mouse turned left about 80% of the time, matching the probability of the food's placement. This behavior is considered suboptimal because the chances of finding food are (0.80 * 0.80 + 0.20 * 0.20) = 68% (which is lower than the 80% chance if the mouse always turned left). However, from an ecological perspective, this strategy may not be irrational. If every mouse consistently followed the rational strategy and turned left, the left side would become overcrowded, reducing the overall chance of finding food. Additionally, none of the mice would explore the less abundant but still available food on the right side. Thus, from an ecological standpoint, a strategy that prevents overcrowding and utilizes all available resources can not be seen as irrational (Mousavi & Kheirandish, 2014). Similar patterns are observed in poker or chess, where players sometimes deviate from the optimal move to surprise their opponents and disrupt their preparations. While these choices might seem suboptimal in isolation, their rationale becomes apparent when the larger context is considered, revealing the limitations of traditional optimality criteria.

The modern-day term 'rationality' originated from the Latin word 'ratio,' which means reason (Page, 2022). Colloquially, a rational decision is one that is reasonable or justifiable, taking into account the significant influence of context in determining what constitutes rationality. However, the contemporary foundation of rationality is grounded in efficient mathematics, supported by principles such as completeness and transitivity (Baron, 2004; Schoemaker, 1982; Von Neumann & Morgenstern, 1944). This mathematical approach allows researchers to theorize and objectively evaluate the quality of individual choices. However, this approach may not always capture the quality of overall choices due to overlooking cognitive and environmental circumstances (Lazear, 2000).

In my doctoral thesis, I investigated the limits of rationality and explored how context plays a critical role in defining rational behavior. The goal was to establish a relationship between the psychological plausibility of individuals' behavior and its ecological effectiveness. Through the thesis, consisting of six empirical studies published in three peer-reviewed journal articles, I explored the idea that individual choices could violate normative principles, yet they can be reasonable. Furthermore, I also explored whether more skilled individuals (i.e., more numerate individuals), due to the numeric nature of the task structure, could better understand the statistical regularities of the task environment and make more adaptive decisions.

Theoretical Overview

In the real world, making good decisions under risk demands flexibility in reactions to dynamically changing task demands. Consider trekking across the Himalayas, with its steep inclines and rapidly changing weather. In this scenario, a trekker must be thoroughly equipped to handle the cold and snow. Conversely, the same individual would opt for a lighter load when navigating the trails of Scotland. Despite the similarities in temperature and climate between these regions, the Himalayas' isolation and ruggedness present a unique set of challenges. Thus, self-sufficiency becomes crucial, and the availability of necessary gear determines the level of preparedness to complete the journey safely. This scenario underscores the importance of evaluating preparedness based on the specific environmental conditions of the trek. Similar but less consequential examples are present in our daily lives as well. For example, if one's goal is to make a good impression during an interview for a desired job, deciding to take an umbrella on a cloudy day is straightforward, and deliberation on its cost and benefit is unnecessary. However, if the personal goal is to have a good time with an old friend, one may seriously consider whether carrying the cumbersome umbrella is worthwhile, accepting the risk of leaving it in a restaurant or getting wet. In other words, even though getting wet or losing an umbrella is not optimal, due to its trivial nature, the outcome is good enough given the context. Both examples emphasize the importance of considering environmental constraints before evaluating the quality of a decision.

Our decisions are influenced not only by environmental constraints but also by our cognitive limitations (Simon, 1990b). For example, finding the best apartment or parking spot can be time-consuming and mentally taxing, especially when the decision maker needs to gather and use information sequentially and iteratively. In a high-demand rental market like London, searching extensively for a property can come with significant costs. One may have to choose between settling for a "good enough" property or continuing the search for a better place. However, continuing the prolonged search might result in losing the current "good enough" apartment without finding a better alternative. In such dynamic and constrained environment, brute-force maximization may lead to inferior outcomes.

The examples above highlight the importance of assessing the quality of our judgments, taking into account both the cognitive and environmental constraints present when making decisions. Formally, the Expected Value (EV) maximization model (i.e., a sum of future outcomes multiplied by the probability of their occurrence) serves as a benchmark for making optimal choices under risk (assuming that a decision maker aims to maximize their rewards). This approach stems from the development of neoclassical theories that assess the optimality of decisions and act as a reference point for the construction of contemporary descriptive theories. (Thaler, 2018). However, since the St. Petersburg paradox described by Bernoulli (Bernoulli, 1954), the EV maximization principle has been challenged as a valid positive model. In response, alternative expectation models have been proposed, all of which follow the basic mathematical framework of the EV maximization model. These alternative models consider actors such as the maximization of expected utility (Von Neumann & Morgenstern, 1944), maximization of subjective value (Tversky & Kahneman, 1992), satisfying (Simon, 1990b), aspirations (Lopes, 1987), or feelings (Loewenstein et al., 2001) as pivotal factors that motivate choices and shape the human decision-making process. Despite the advantages of these descriptive models, the EV model still serves as a reference point for making optimal choices under risk from a formal perspective.

3.1 Good Decisions and Individual Differences

The ability to understand and use numerical data effectively, a construct known as statistical numeracy, is essential for making well-informed decisions (Ancker & Kaufman, 2007; Cokely et al., 2018; Patalano et al., 2015; Reyna & Brainerd, 2007; Schwartz et al., 1997). Since the introduction of the first psychological tests measuring numeracy, research in this field has grown rapidly (Garcia-Retamero et al., 2019; Nelson et al., 2008; Reyna et al., 2009). Studies consistently show that individuals with high numeracy make decisions that are more aligned with EV maximization models and are less influenced by cognitive biases compared to those with low numeracy (Liberali et al., 2012; Peters & Levin, 2008; Reyna & Brainerd, 2007).

Aligned with these findings, in a longitudinal investigation involving a substantial cohort of Dutch adults, Estrada-Mejia et al. (2016) found that numeracy is a crucial determinant of wealth accumulation trajectories over time. In other words, more numerate individuals, over time, accumulated more wealth, whereas individuals with low numeracy tended to recede wealth. The authors estimated that, on average, a one-point increment in numeracy score is associated with a five percent rise in personal wealth. One potential explanation for the increased wealth accumulation among highly numerate individuals is that more numerate individuals consistently make more choices that are consistent with the EV maximization strategy (Cokely & Kelley, 2009; Millroth & Juslin, 2015; Mondal, 2021; Sobkow et al., 2020). Additionally, more numerate individuals are more sensitive to variations in EVs and tend to spend more time on decision-making processes compared to less numerate individuals (Jasper et al., 2013; Mondal, 2021; Peters & Bjalkebring, 2015; Traczyk, Sobkow, et al., 2018). Furthermore, metacognitive factors (such as deliberation) also play an important part in aiding the decision process of highly numerate individuals in making better monetary decisions.

The advantage of superior numeric skills is not restricted to the financial domain only. Existing literature from the field of medical decision making also corroborated earlier findings and argued that numeracy is a crucial predictor of superior outcomes in the health and medical decision making domain as well. A systematic review conducted by Garcia-Retamero and Cokely (2017) reported that numeracy is strongly correlated with how accurately individuals perceive the risks and benefits of health-related behavior. For example, less numerate individuals often take more time before seeking medical attention, which increases their risk of suffering from severe diseases and, at the same time, reduces their chance of detecting diseases at an early stage (Apter et al., 2006; Petrova et al., 2017). Furthermore, less numerate individuals often avoid asking treatment-related questions to their doctors (Paasche-Orlow & Wolf, 2007), negatively affecting the quality of medical and shared decision making between doctors and patients (Galesic & Garcia-Retamero, 2011). In addition, evidence shows that patients with low numeracy often have more difficulty following a complex dosing regimen (Waldrop-Valverde et al., 2010). Overall, the robustness of numeracy as an essential predictor of better health outcomes is unquestionable (Garcia-Retamero et al., 2019).

Several fundamental mechanisms have been identified in exploring the psychological factors that contribute to the variance in decision-making abilities between individuals with high and low numeracy. These mechanisms include: (1) a more linear weighing of outcomes and probabilities according to parameters from Cumulative Prospect Theory (CPT; Millroth & Juslin, 2015; Patalano et al., 2015; Petrova et al., 2014; Traczyk & Fulawka, 2016), (2) a more extended deliberation on the problem (Ghazal et al., 2014; Petrova et al., 2016), (3) elaborate and in-depth processing of information, utilizing heuristics (Cokely & Kelley, 2009), (4) the formation of gist representation of the decision problem (Broniatowski & Reyna, 2018; Reyna & Brainerd, 2008), which facilitates a more precise assessment of the accuracy of the decision made (Barrafrem et al., 2021; Garcia-Retamero et al., 2015), and (5) the integration of significant emotional information (for instance, the fear of adverse outcomes from decisions) into the decision-making process (Peters, 2012; Traczyk & Fulawka, 2016; Traczyk, Lenda, et al., 2018).

Among many theoretical accounts on the psychological mechanism underlying numeracy, the Fuzzy Trace Theory (FTT) can potentially make the most diverse and robust predictions regarding the difference in performance between high and low numeracy individuals (see Broniatowski & Reyna, 2018; Reyna & Brainerd, 2008, for a detailed comparison between competing models). FTT argues that individuals simultaneously encode three distinct kinds of memory representation of the same numerical information: verbatim representations, ordinal gist representation, and categorical gist representation (Reyna & Brust-Renck, 2020). FTT posits that more skilled decision makers are able to make better choices due to employing a more deliberate metacognitive decision process that helps them to create and use a verbatim representation of the choice problem instead of relying on non-precise gist representation. Meanwhile, less numerate individuals often rely more on gist representation and spend significantly less time on each decision. Put simply, more numerate individuals, due to their numeric competency, are able to better understand the statistical regularities of the environment, providing them with the sufficient context needed to make good decisions (i.e., normatively superior decisions).

3.2 Skilled & Adaptive Decision Makers

In theory, following a normative decision strategy, on average, leads to good decisions. However, the real world is far from perfect, as are the decision makers who operate within it. It is characterized by uncertainty, complexity, and constraints. Consequently, the complexity of the environment, coupled with the limited computational ability, restricts decision-makers from implementing a brute-force optimization process prescribed by the normative theory (Simon, 1990a). The theory of resource rationality was proposed to provide a more comprehensive understanding of human cognition by integrating the limitations of computational resources with principles of rational behavior. Traditional models of rationality often assumes that individuals have unlimited cognitive resources and access to perfect information, which is unrealistic. The theory of resource rationality argues that people make decisions based on the optimal use of their cognitive resources, such as time, attention, and computational capacity. This theory is a bridge between the notions of bounded rationality and fully rational decision making (Lieder & Griffiths, 2020). This approach acknowledges human cognitive limitations and suggests that individuals use their resources efficiently to achieve the best possible outcomes within these constraints. Instead of seeking the absolute best decision, individuals often settle for "good enough" options (satisfying) to conserve resources. They employ heuristics and simplified models as mental shortcuts to facilitate quicker and less resource-intensive decision making. These strategies are adaptive responses to the environment, developed through experience and tailored to specific contexts (Krueger et al., 2024). Resource rationality involves balancing the trade-offs between the costs of cognitive resources and the benefits of improved decision accuracy. The goal is to maximize overall utility by considering both outcomes and resource costs (Bhui et al., 2021).

The heuristics literature, however, takes a different approach from the theory of resource rationality (Gigerenzer, 2008). Heuristics are characterized by cognitive shortcuts or rules of thumb that enable decision makers to solve problems quickly and efficiently. It simplifies complex decision problems by approximating optimal strategies (Gigerenzer, 2008) to a satisfactory level while reducing the amount of cognitive effort and information needed to arrive at a suitable conclusion (Gigerenzer & Gaissmaier, 2011). While heuristics can lead to biases and errors, more often than not, they often approximate optimal choices sufficiently well and sometimes even outperform them. For instance, the work by DeMiguel et al. (2009) revealed that a straightforward and economical approach of dividing investment funds equally ($\frac{1}{n}$) resulted in greater financial gains within a shorter period compared to the optimal asset allocation strategy introduced by Harry Markowitz[1] (Markowitz, 1952). This example illustrated that following a simple strategy could result in better overall performance than a complex optimal policy in certain situations. An excellent example of such a fast and frugal heuristic that works seamlessly in the real world is the gaze heuristic. It involves a catcher keeping a constant gaze angle to accurately intercept a

[1]Harry Markowitz was awarded the 1990 Nobel Prize in Economics for his groundbreaking contributions to portfolio management and risk evaluation.

ball (McLeod & Dienes, 1996). In other words, a catcher needs to increase or decrease their running speed depending on the angle between them and the ball so that the angle of the gaze remains constant (Gigerenzer, 2007). The first formal discovery and application of the gaze heuristic happened during the Second World War (Hamlin, 2017). Sir Henry Tizard, with substantial help from E. O. Grenfell, the commander of an RAF station, created the interception control system[2] that proved to be remarkably effective in intercepting Luftwaffe bombers with high accuracy (Kirby, 2003). The knowledge of the gaze heuristic proved to be very important for the Allied forces because their German counterparts were unaware of such an effective heuristic. Due to this reason, despite the technological superiority of the German radars, the Luftwaffe was at a significant disadvantage, and the success of their bombing campaigns proved to be limited (Jones, 1978).

The example illustrated how specific knowledge increased the average proficiency of a force, reduced mistakes due to the simplicity of the strategy, and contributed towards a more favorable outcome in the long run. Similar examples of expertise influencing performance have also been found in other areas as well. Expert chess players (i.e., grandmasters) could recall virtually all of the positions that were presented to them compared to weaker players. However, when those positions were randomly put together, the performance of the expert chess players was no better than the weaker players (de Groot, 1965). Grandmasters could effectively recall each position due to their understanding of the relationship between chess pieces. Expert chess players do not have a larger memory capacity, nor do they consider more moves; rather, they know which information is more pertinent, where to look, and which information to prioritize (Chase & Simon, 1973; Cooke et al., 1993; Ericsson & Pool, 2016; Schneider et al., 1993). Such abilities help more skilled decision makers to better understand the statistical regularities in the decision environment and adjust their strategies to meet the demands of different tasks. They do this by finding a balance between effort and accuracy, shifting their focus from the depth of evaluation to the breadth of evaluation (Payne & Bettman, 2004; Payne et al., 1993). In the thesis, I investigated whether the adaptive qualities of highly skilled decision makers could be generalized to those who are highly proficient in dealing with numbers (i.e., more numerate individuals) when dealing with numeric problems.

[2] Tizard introduced the concept of Tizzy Angle where interception of a bomber was done using equal angles method. For more details, see DeGering (2018).

Overview of the Studies

4.1 Objective

The theoretical overview indicates that skilled decision makers better understand the context in which they operate, and more importantly, skilled decision makers, compared to less skilled decision makers, take into account the associated environmental and cognitive limitations to make superior decisions. In this thesis, I extended this finding to the field of numeracy and adaptive decision making using six empirical studies and one simulation study documented over three published articles (see Chapter 5, 6, 7 for a summary, or see Chapter 10.1, 10.2, 10.3 for the published manuscript[1]).

The terms "context" and "adaptive decision making" represent quite expansive and sometimes confusing notions in the field of psychology. In the thesis, the term "context" signifies the constraints (both cognitive and environmental) associated with the task structure presented to the participants. "Adaptive decisions" is a term that is defined for each study separately, but all these definitions share a common thread. That is, a single decision cannot be adaptive. Any group of decisions can be adaptive if it collectively outperforms the existing normative prediction or matches it without utilizing a similar amount of resources. In other words, decisions that balance opportunity cost and maximize reward or minimize accumulated loss.

The main goal of the thesis was to investigate the relationship between numeracy and adaptive decision making. Initially, I replicated the finding of Traczyk, Sobkow, et al. (2018) where the authors argued that more numerate individuals, compared to less numerate individuals, better adaptively modulated their decision strategy following the changes in relative difference in the payoff structure (High-payoff vs. Low-payoff condition). I successfully replicated the result and demonstrated that the reported relationship is robust and replicable (see Chapter 5 for a summary or see Chapter 10.1 for the published manuscript). Although I successfully replicated the Traczyk, Sobkow, et al. (2018) study, I identified a few limitations that could affect the overall conclusion. Therefore, in the second publication, I planned to mitigate limitations (i.e., lack of trade-off between strategies, asymmetry in the distribution of absolute difference in EV) of earlier Traczyk, Sobkow, et al. (2018) study and probed the effect of adaptive modulation in decision strategy. I found that payoff conditions alone did not invoke adaptive strategy selection, irrespective of numeracy levels. I identified

[1]For the sake of brevity and continuity, I have labeled each empirical study as they were presented in each publication.

that, contrary to the earlier result, numerate individuals were better at maximizing expected reward following the absolute difference in value between options and not the relative difference in value embodied by the two payoff conditions (see Chapter 6 for a summary or see Chapter 10.2 for the published manuscript). In the previous study, I dissociated the relationship between payoff conditions and adaptive strategy selection. In the third and last study, I constructed a more simplified environment, where making adaptive decisions is simple yet contrary to existing normative principles of decision-making literature, to examine whether higher numeracy enables decision makers to make better adaptive decisions. I confirmed my primary hypothesis and demonstrated that individuals with higher numeracy adaptively modified their exploration strategies in response to the alterations in the task environment. Specifically, they engaged in more dynamic and flexible information sampling and employed smart search strategies to make recurringly suboptimal choices that were adaptively rational (see Chapter 7 for a summary or see Chapter 10.3 for the published manuscript).

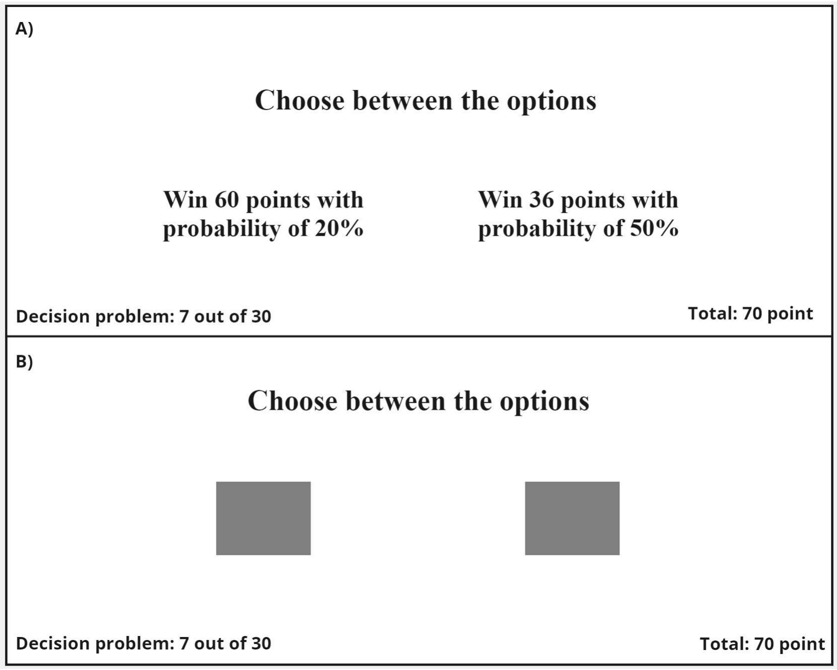
4.2 General Methodology

In all empirical studies, participants were presented with choice problems alongside a scale that measured the numeric proficiency of decision makers. In Chapter 5 and Chapter 6, participants were presented with the choice problems using the Decision from Description (DfD) paradigm. Whereas, in Chapter 7, choice problems were presented to participants following the Decision from Experience paradigm (DfE).

In the DfE tasks, individuals were required to make choices based on outcomes they had personally observed through repeated exploration. This process relied heavily on the accumulation of personal experiences by repeated sampling of both options to understand the underlying probability structure. Because individuals directly interacted with the probabilities and outcomes over time, they often exhibited tendencies like underweighting of rare events (Hertwig & Erev, 2009). In contrast, the DfD task involved participants being provided with explicit descriptions of the probabilities and outcomes associated with different options. Here, decisions were based on provided statistical information rather than personal experience.

In the thesis, the presentation of outcome and probability information for both options was straightforward for the DfD paradigm. As captured by Fig. 4.1A, participants were explicitly informed about the outcome and corresponding probabilities of options. For the DfE task paradigm, as captured by 4.1B, participants were presented with two boxes symbolizing binary two-outcome gambles with an unknown payoff and probability distribution for the DfE task paradigm. Upon selecting a gamble, an outcome was drawn randomly from a given distribution and shown for 400 ms. Participants could decide by themselves which distribution they want to sample from, when to switch between the gambles, and when to terminate exploration. After participants finished sampling and were ready to make a decision, they had to indicate which gamble they preferred by clicking the “Done” button below the boxes and then selecting a gamble by clicking on the corresponding box. Regardless of whether participants were presented with the DfE or DfD paradigm, feedback on

Figure 4.1: A representative example of the procedural differences between the two paradigms. A) Decision from description task paradigm. B) Decision from experience task paradigm.



their choice was provided immediately. In addition, information on their total current gain was presented in the screen's bottom-right corner, given that the study was performance contingent. I recorded participants' preferences, deliberation time, the number of decisions, the number of samples while exploring the gambles, time spent on exploration, and calculated the switching ratio (i.e., the ratio between the number of actual switches between gambles and the possible number of all switches given a total number of drawn samples).

Besides the choice problems, each participant was presented with a questionnaire that assessed their numeric abilities. To measure statistic numeracy, I used the Berlin Numeracy Test (BNT; Cokely et al., 2012). The BNT is a standardized psychometric tool designed to accurately assess a person's numerical abilities. This includes their understanding of statistics, ability to comprehend risk, and grasp of probability concepts. The BNT consists of mathematical tasks of varying difficulty. For example:

"Imagine we are throwing a five-sided die 50 times. On average, out of these 50 throws, how many times would this five-sided die show an odd number ?"

I employed the computerized 4-item BNT version, with items appearing one after the other in a predetermined sequence. The total of accurate responses on the BNT was tallied to determine the numeracy score, which ranged from 0 to 4.

4.2.1 Ethics

The experimental procedures of all studies reported in the thesis were approved by the Ethical Review Board at SWPS University. All participants provided their informed consent before taking part in the studies. The aim of the study was explicitly explained to the participants, and no deception was used in the experimental design. Participants were repeatedly informed that they could quit the study without any consequence. Furthermore, participants were transparently informed about the payment terms before they initiated the study, and there were no hidden conditions regarding payment. Lastly, the research maintained strict confidentiality protocols, ensuring that only the project's research team could access participants' personal information. However, this data was anonymized prior to collective analysis.

4.2.2 Open Science Practice

My thesis adheres to the principles of open science, as I believe that transparency and reproducibility are crucial for advancing science. Additionally, all studies reported in the thesis were pre-registered. By making data (anonymous data), methodologies (i.e., experimental procedure, choice problems, scales, code for the experiment), and results (i.e., data used for analysis, R code, JASP files) publicly available, I enable other scholars to examine, validate, and extend my findings, promoting cooperation and speeding up scientific progress.

In the following, I have listed the Open Science Framework (OSF) repository links where one can find both materials and preregistration for all the studies reported in the published articles.

Chapter 5: Replication of Traczyk et al.(2018)

Pre-registration: https://osf.io/pwb53\ **Resources:** https://osf.io/cje9b/

Chapter 6: Conditionality of adaptiveness

Pre-registration: https://osf.io/5zmws (Experiment 1) and https://osf.io/zrksp (Experiment 2)
Resources: https://osf.io/p8av4/ (Experiment 1) and https://osf.io/67hwc/ (Experiment 2)

Chapter 7: Recurring Suboptimality

Pre-registration: https://osf.io/k7tfx (Study 1) and https://osf.io/u59fc (study 3)
Resources: https://osf.io/56xfa/

Summary: Replication of Traczyk et al.(2018)

Publication: Mondal, S. (2021). Sensitivity of numerate individuals to large asymmetry in outcomes: A registered replication of Traczyk et al.(2018). *Decyzje*, (*35*), 5-26. https://doi.org/10.7206/DEC.1733-0092.150a.

5.1 Introduction

Every day, we encounter statistical information that influences our decision making, whether it is deciding on a tip amount at a restaurant, investing in funds, or purchasing groceries. While many of our decisions may seem insignificant, there are times when our choices can have a major impact. In these crucial moments, the quality of our decisions depends on our level of expertise in the relevant field. For instance, a statistician would be less optimistic about winning the lottery compared to someone without knowledge of the field. Expertise significantly shapes our preferences as well as expectations.

Traczyk, Sobkow, et al. (2018) tested the above conjecture using a series of 24 two-outcome choice problems, where choice problems were divided into the High-payoff condition (EV ratio is around 5) and the Low-payoff condition (EV ratio is around 1). Decision problems were considered less important in the Low-payoff condition because participants would earn relatively similar payoffs regardless of their choices. However, choice problems in the High-payoff condition were considered to be more important because selecting a gamble with a higher EV, on average, would lead to a significantly higher payoff. The authors observed that individuals with higher objective numeracy maximized EV and made choices consistent with the predictions of Cumulative Prospect Theory (CPT; Tversky & Kahneman, 1992) in the High-payoff condition. However, in problems where the EV ratio between gambles was low, and the potential outcomes were comparable, highly numerate participants adapted their strategy and made choices consistent with the predictions of the Priority Heuristic (PH; Brandstätter et al., 2006). This result demonstrates that more numerate individuals, compared to less numerate individuals, were more likely to follow an adaptive decision strategy and switch between effortful compensatory strategy to a heuristics strategy following the change in relative difference in reward between options (i.e., High-payoff vs. Low-payoff). This was one of the very first reported findings showing that individuals with better numeracy skills tended to make more adaptive decisions. Therefore, it was of great interest for the field of numeracy and adaptive decision making to see if this result could be replicated or not.

In this study, I planned to replicate the findings of Traczyk, Sobkow, et al. (2018) regarding the impact of statistical numeracy on decision making. Specifically, I aimed to confirm whether individuals with high statistical numeracy were more likely to make choices consistent with the predictions of CPT/EV compared to those with low statistical numeracy in the High-payoff condition but not in the Low-payoff condition.

Being a replication study, I used a very similar study procedure compared to the original study. Data was collected from seventy-three adult volunteers ($M_{age} = 27$ years) for a half-hourly compensation of £4.00 GBP. Participants were presented with 24 choice problems (divided between High-payoff and Low-payoff condition), International Cognitive Ability Resource (ICAR, a measure of fluid intelligence; Condon & Revelle, 2014), Berlin Numeracy Test (BNT, a measure of objective numeracy; Cokely et al., 2012), and Subjective Numeracy Scale (SNS, a measure of subjective numeracy; Fagerlin et al., 2007) in a random order.

5.1.1 Choice problem

I used almost the same 24 choice problems (except one[1]). These problems were specifically chosen to differentiate between the strategy predicted by the heuristic strategy (PH) and the expectation models, characterized by the weighting and summing operation (CPT/EV). For example,

Gamble A:	\$5.40 with 29%;	\$0 with 71%
Gamble B:	\$9.70 with 17%;	\$0 with 83%

PH predicts that a decision maker will likely select Gamble A due to the larger gap in probability for the minimum gain options (0.71 compared to 0.83) exceeding 10% of the probability scale. In contrast, CPT with standard parameters from (Tversky & Kahneman, 1992) predicts that a decision maker will prefer Gamble B because of its greater CPT value (i.e., 1.38 vs. 1.78). Under the current experimental procedure, CPT predictions were to be the same as EV maximization strategy[2]. Therefore, whatever participants decided, it would match with either PH theory's prediction or would resemble EV maximization strategy/CPT theory's prediction (for more elaboration, see Pachur et al., 2013).

[1] One choice problem differed from the original study because of inappropriate translation of research materials from Polish to English.

[2] CPT prediction is aligned with EV maximization strategy for the current set of choice problems. All the gambles CPT predicts also have higher EV (i.e., EV of 1.57 vs. 1.65 for Gamble A vs. Gamble B). Therefore, when participants choose any gamble predicted by CPT, it means choosing a gamble with a higher EV value.

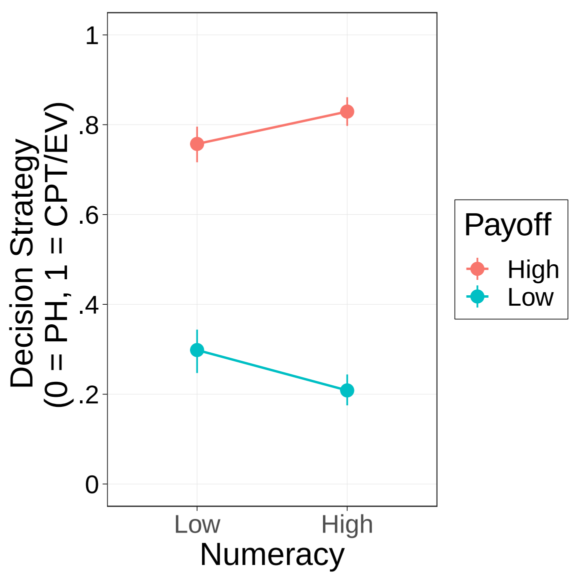
5.1.2 Differences Between the Original and Replication Study

Unlike the original study, the current study did not use the Need for Cognition Scale (NCS; Cacioppo & Petty, 1982) and Raven's Advanced Progressive Matrices (RAPM; Raven, 2000). Instead, the current study used International Cognitive Ability Resource (ICAR; Condon & Revelle, 2014) as a replacement for RAPM. Additionally, the mode of instruction between the original and the current replication study was different. The original study was in Polish, while the current replication is in English. As a consequence, participants in the original study were mainly from Poland, but anyone proficient in English can partake in the replication study.

5.2 Result & Discussion

The present study examined whether people with higher statistical numeracy, in comparison to people with lower statistical numeracy, strategically employed a more effortful choice strategy to make adaptive choices when the choice problems are meaningful.

Figure 5.1: Decision strategy is plotted across two payoff conditions as a function of numeracy levels. Here, 0 = PH refers to choices consistent with the Priority heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.



According to the current findings, participants more frequently used the compensatory decision strategy embodied by CPT/EV in meaningful problems ($M = 0.80$, $SD = 0.20$) compared to Low-payoff problems ($M = 0.20$, $SD = 0.20$). Further analysis, using the Mann-Whitney U-test, showed a significant difference in choice strategy between individuals with higher and lower numeracy in High-payoff problems ($U =$

118852.50, p = .006, r_b = 0.07). On the other hand, in the Low-payoff condition, highly numerate individuals tended to switch to a non-compensatory policy that resembled the predictions from PH more often than individuals with lower numeracy skills (U = 72242.50, p = .003, r_b = -0.09). This shift in strategy was observed in all participants but was more pronounced in those with higher numeracy, indicating their ability to adapt to changes in the payoff structure. These findings successfully replicated the results of the original study.

This study showed that individuals with high statistical numeracy tended to use more energy-intensive decision-making strategies when faced with important choices compared to individuals with low statistical numeracy. However, when the decision was less important, the more numerate individuals used quicker, less effortful heuristic strategies to save time and mental effort. Importantly, the current study illustrated that objectively numerate decision makers were more sensitive to changes in relative differences in the reward structure and could better modulate their strategy to adapt to the changes in the task environment. Taken together, the result from the current study successfully replicated the effect of numeracy and adaptive decision making.

Summary: Conditionality of Adaptiveness

Mondal, S., & Traczyk, J. (2023). Conditionality of adaptiveness: Investigating the relationship between numeracy and adaptive behavior. *Journal of Economic Psychology*, *96*, 102611. /https://doi.org/10.1016/j.joep.2023.102611. (Impact Factor: 3.5)

6.1 Introduction

The replication study of Traczyk, Sobkow, et al. (2018) conclusively demonstrated that more numerate individuals could strategically invest resources to follow adaptive decision strategy. When facing decisions with significant consequences and potentially higher rewards, more numerate individuals tended to make decisions aligned with normative decision-making theories such as CPT and EV. However, when facing trivial decisions with similar potential rewards, more numerate individuals were more likely to rely on heuristic decision-making strategies compared to less numerate individuals.

However, the previous study suffered from some limitations. This study was designed to address the limitation of the replication study and extend this line of research. Choice problems used in earlier studies were not well controlled. Contrary to the conclusion, payoff was not the only varying factor between the High- and Low-payoff conditions. The absolute difference in expected values (AED) between the two options had a skewed distribution across the two payoff conditions —that is, choice problems in the Low-payoff condition consistently had lower absolute differences in the expected values compared to the High-payoff condition. Furthermore, the difficulty level between the two payoff conditions was also inconsistent (Pachur et al., 2013). As a result, adaptive modulation in decision strategy between two payoff conditions could not only be attributed to varied EV ratios embodied by payoff conditions. Put differently, it was not clear whether payoff conditions alone could provide sufficient context necessary for highly numerate individuals to make better adaptive modulation in decision strategy compared to less numerate individuals. In two empirical studies (i.e., Experiment 1 & Experiment 2), I investigated adaptive strategy selection between highly numerate and less numerate individuals with more evenly distributed choice problems.

6.2 Experiment 1

In the first experiment, I expected that highly numerate individuals would, on average, make more EV-consistent choices compared to less numerate individuals in the High-payoff condition (i.e., when the relative difference in reward between two options is significantly large) but not in the Low-payoff condition (i.e., when the relative difference in reward between two options is comparable) by recognizing the relative difference in payoff distribution across the two payoff conditions.

I presented each participant with 36 High-payoff and 36 Low-payoff choice problems to record their preference and reaction time. In addition, the BNT scale was used to measure their objective numeracy score. The presentation of choice problems and the numeracy scale was counterbalanced. I recruited ninety-five volunteers to participate (M_{age} = 26.32 years, SD_{age} = 6.81) in an online study for a half-hourly compensation of 2.56 GBP. As a bonus, participants were informed that for every 1000 points, they would receive an additional 1 GBP on top of the flat fee.

Choice problems

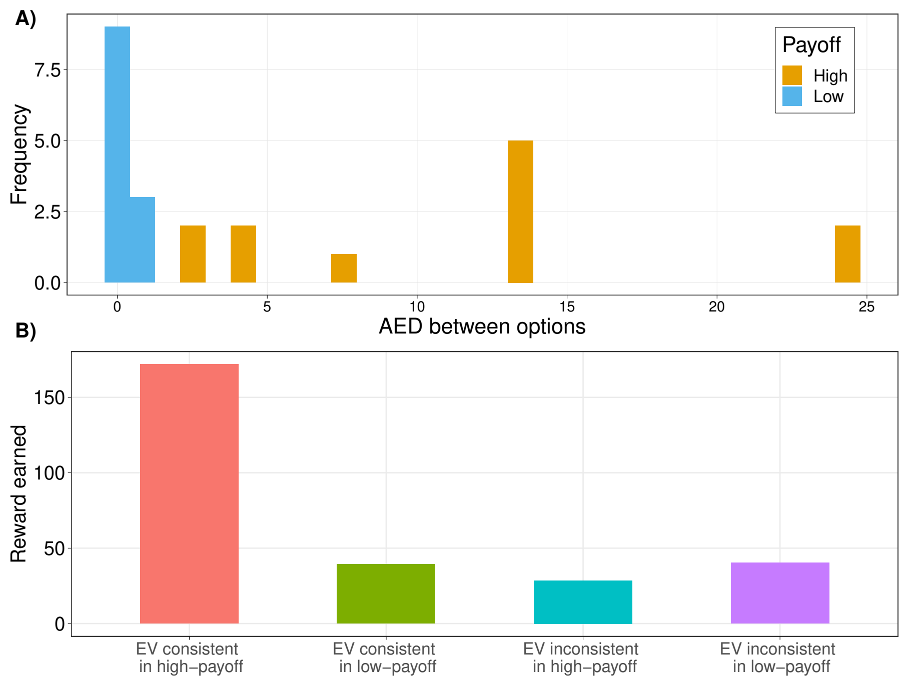
I developed 72 new binary choice problems consisting of two-outcome gambles distributed evenly between the High- and Low-payoff conditions in the gain domain. The absolute-EV-Difference (AED) between options was not controlled in the earlier choice problems used in (Traczyk, Sobkow, et al., 2018). AED between options was consistently higher for choice problems in the High-payoff condition, whereas AED between options was always lower for choice problems in the Low-payoff condition (see Fig. 6.1A). This skewness in AED distribution, as captured by Fig. 6.1B, resulted in an asymmetric trade-off between EV-consistent and EV-inconsistent choices.

In the current study, the purpose was to create choice problems controlling the difference in variance, AED between options, and asymmetric trade-off across two payoff conditions to let only the EV ratio vary between the two conditions (see Table 6.1). This design ensured that, regardless of the payoff condition, participants would earn significantly less if they decided to make EV-inconsistent choices compared to EV-consistent choices. Every measure was taken to ensure that only the EV ratios varied between the two payoff conditions for all participants. Specifically, the choice problems were explicitly selected to distinguish between the strategy predicted by the heuristic strategy (PH) and the weighting and summing operation characterized by expectation models (CPT/EV).

6.2.1 Results & Discussion

This study aimed to investigate the impact of adaptive strategy selections with more controlled choice problems. Here, adaptive strategy selection refers to the modulation in decision strategy (i.e., the use of compensatory decision strategy in High-payoff problems and the use of non-compensatory heuristic decision strategy in Low-payoff problems). To examine adaptive modulation in strategy, I used the independent sample Mann–Whitney U-test to compare EV consistency in the High-payoff and

Figure 6.1: Distribution of Absolute EV Difference (AED) and possible earning is plotted across two payoff conditions for choice problems taken from (Traczyk, Sobkow, et al., 2018). A) The distribution of Absolute EV Difference (AED) between options is plotted across the two payoff conditions. B) Possible earning of reward is plotted as a function of different choice strategies. Here, EV-consistent refers to choices consistent with the EV maximization model, and EV-inconsistent indicates choices inconsistent with the EV maximization model.



Low-payoff conditions between high and low numeracy groups. The results suggested that more numerate individuals followed EV-consistent choices, consistent with the hypothesis and earlier results, significantly more times, $U = 1459$, $p < .001$, $r_b = .45$, in the High-payoff condition ($M = 0.90$, $SD = 0.13$) compared to less numerate participants ($M = 0.82$, $SD = 0.13$). However, as Fig. 6.2 suggests, this trend continued, inconsistent with the hypothesis and earlier results, in the Low-payoff condition as well. Highly numerate individuals followed EV-consistent choices ($M = 0.85$, $SD = 0.12$), significantly more often, $U = 1412.5$, $p < .001$, $r_b = .40$, in the Low-payoff condition compared to less numerate individuals ($M = 0.77$, $SD = 0.13$).

The result indicated that the payoff condition alone was not sufficient for more numerate individuals to make adaptive strategy selections. In other words, when AED between options was controlled, the presence of the High-payoff and Low-payoff conditions together necessarily did not initiate adaptive strategy selection, regardless of participants' numeracy. In addition, the magnitude of difference in EV-consistent choices between the two payoff conditions, Compared to the replication study, was also relatively small.

This result led me to two conclusions. First, modulation in EV consistency was

Table 6.1: Composition of the newly developed 72 choice problems. High-payoff and Low-payoff conditions constitute EV ratios of 5-6 and 1-2, respectively. Large AED refers to the AED difference of 33.3-50; Medium AED refers to the AED difference of 16.6-33.2; Small AED refers to the AED difference of 0-16.6.

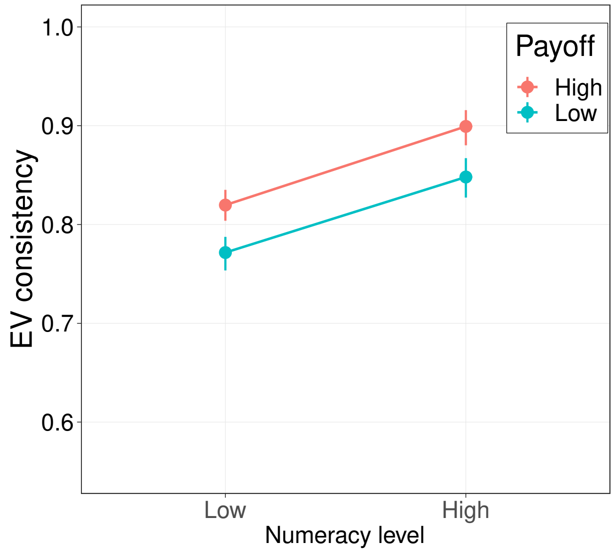
AED	Payoff	Total Number	Representative example	AED between options	EV ratio
Large	High	12	A) 9% chance to receive 93 points B) 63% chance to receive 70 points	35.73	5.27
Large	Low	12	A) 55% chance to receive 92 points B) 99% chance to receive 90 points	38.5	1.76
Medium	High	12	A) 8% chance to receive 59 points B) 54% chance to receive 45 points	19.58	5.15
Medium	Low	12	A) 33% chance to receive 61 points B) 79% chance to receive 50 points	19.37	1.96
Small	High	12	A) 4% chance to receive 58 points B) 99% chance to receive 13 points	10.55	5.55
Small	Low	12	A) 81% chance to receive 39 points B) 31% chance to receive 71 points	9.58	1.44

informed by the understanding that the relative difference in value was significantly dissimilar across the two payoff conditions (i.e., EV ratio 1–2 in the Low-payoff condition compared to EV ratio 5–6 in the High-payoff condition), but the magnitude of that effect was comparatively small. Second, participants did not perceive the relative difference in value, and the presence of two payoff conditions together had no significant effect on the changes in decision strategy. Instead, the change in EV consistency across the two payoff conditions was largely due to participants' response to the absolute difference in value (i.e., AED between options) regardless of the relative difference in value.

6.3 Experiment 2

In the second experiment, I planned to eliminate one of the two interpretations of the previous result. In Experiment 1, participants were presented with choice problems from both payoff conditions to provide them with the context of significant relative differences in reward between options. In the second study, I presented only one payoff condition to each participant. Hence, if there was no meaningful difference in EV consistency between the two payoff conditions, then one can, with sufficient confidence, conclude that the change in EV consistency occurred due to participants' recognition of the relative difference in value between the two payoff conditions (i.e.,

Figure 6.2: EV consistency plotted as a function of numeracy score across two payoff conditions.



EV ratio of 1–2 in the Low-payoff condition compared to EV ratio of 5–6 in the High-payoff condition). Otherwise, the change in decision strategy across two payoff conditions was due to participants' response to the absolute difference in value (i.e., AED between options).

Unlike Experiment 1, participants were presented with either 36 Low-payoff or 36 High-payoff choice problems together with a numeracy scale in a random order. Two hundred and forty-eight volunteers ($M_{age} = 27.7$ years, $SD_{age} = 8.92$) participated in an online study for a half-hourly compensation of 2.5 GBP. As a bonus, in the Low-payoff condition, participants were told that for every 1000 points, they would receive an additional 0.80 GBP on top of the flat fee. However, in the High-payoff condition, participants were told that for every 500 points, they would receive an additional 0.65 GBP on top of the flat fee. The difference in bonus payment between the two conditions was a reflection of the earning difference between the two payoff conditions.

6.3.1 Results & Discussion

I used the independent groups Student's equivalence test to examine the non-inferiority of means between the two payoff conditions (Lakens et al., 2018). Employing the non-inferiority test enabled me to prove that something does not exist. In other words, the non-inferiority test paradigm permitted me to show that the mean of one group is neither meaningfully larger nor smaller than the other group. Instead of requiring the means of the two groups to be identical, the standard practice is to define an acceptable range of closeness (δ) within which the groups are considered sufficiently similar.

I used a three-point scale instead of a single absolute value to establish the degrees of closeness (δ) between the two conditions. When $\delta < 5\%$ (definitive non-inferiority), the non-inferiority test was non-significant, $t(246) = 1.166$, $p = .878$, given the bounds of -Inf and 0.040 (on a raw scale) with an alpha of 0.05. Similarly, when $\delta < 10\%$ (probable non-inferiority), the non-inferiority test was non-significant, $t(246) = -1.166$, $p = .122$, given the bounds of -Inf and 0.080. However, when $\delta < 15\%$ (anecdotal non-inferiority), the non-inferiority test was significant, $t(246) = -3.497$, $p < .001$, given the bounds of -Inf and 0.120. After comparing the average consistency of expected value (EV) across both payoff conditions in Experiment 1 and Experiment 2, I concluded that the significant anecdotal non-inferiority was not meaningful due to the similarity in distribution and magnitude of modulation in decision strategy across both conditions.

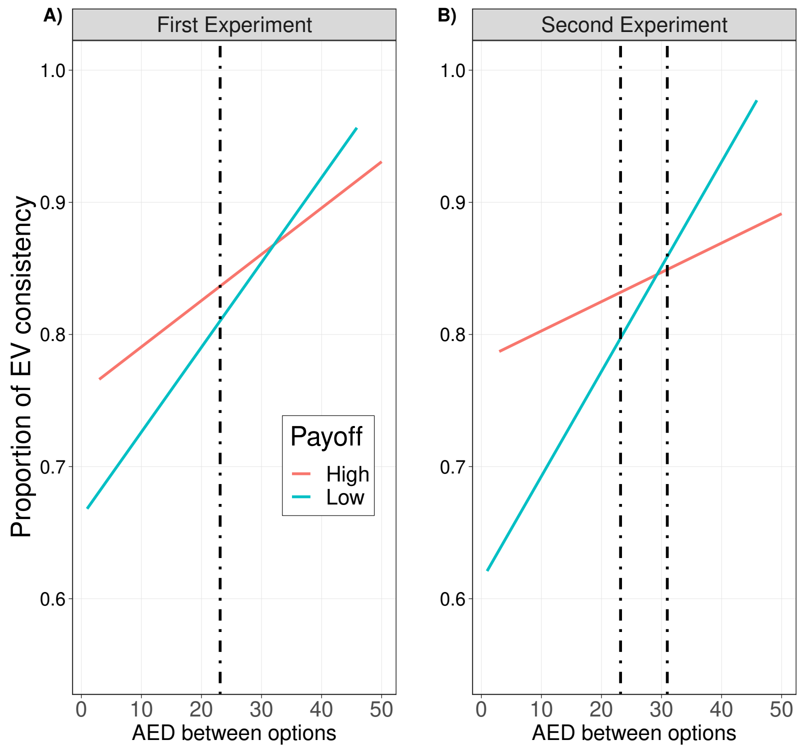
The result indicated that the mean EV consistency in the Low-payoff condition was not non-inferior to the mean EV consistency in the High-payoff condition. In other words, Experiment 2 conclusively indicated that numeric information on the relative differences was insufficient for decision makers to make adaptive strategy selections. This result also highlighted the effect of AED between options across the two payoff conditions. I ran a moderation analysis to explore this effect further and calculated the Johnson-Neyman interval. Results from the moderation analysis suggested that there was a significant interaction, $\beta = -0.034$, $Z = -8.19$, $p < .001$, between payoff and AED distribution on EV consistency. There was no significant direct effect, $\beta = 0.104$, $Z = 1.75$, $p = .08$, of payoff conditions on EV consistency. On the other hand, there was a significant direct effect, $\beta = 0.033$, $Z = 16.28$, $p < .001$, of AED on EV consistency. The current result, as captured by Fig. 6.3, mirrored the result from Experiment 1. Lastly, as Fig. 6.3B suggested, the Johnson-Neyman analysis indicates that when AED is outside the interval of the [23.18, 30.96] points, the slope of payoff difference was significant ($p < .05$).

6.4 Conclusion

Earlier studies emphasized the importance of payoff conditions on adaptive decision-making (Mondal, 2021; Traczyk, Sobkow, et al., 2018). However, the current results from both studies revealed that the payoff conditions alone did not invoke adaptive strategy selection, irrespective of numeracy levels. In earlier studies, multiple factors (i.e., lack of trade-off between strategies and asymmetry in AED distribution) were inconsistent between the two payoff conditions. However, when controlled, I identified that numerate individuals were better at maximizing expected reward following the absolute difference in value and not the relative difference in value embodied by the two payoff conditions. My original contribution is to successfully dissociate the relationship between payoff conditions and adaptive strategy selection.

The current study also highlighted the importance of focusing on absolute values. Almost all modern theories of decision making are built on the idea of relativity (Friedman & Savage, 1952; Tversky & Kahneman, 1992), where preference for an item is judged with respect to other items. The higher-value object is consistently preferred if the difference in value is relatively large between items. However, as the

Figure 6.3: EV consistency of Experiment 1 and Experiment 2 is plotted as a function of Absolute EV Difference (AED) across the two payoff conditions. Here, the dotted line refers to the Johnson-Neyman interval of insignificance. A) In Experiment 1, once AED crosses the dotted line [23.11], there is no significant difference in EV consistency between the two payoff conditions. B) In Experiment 2, there is no significant difference in EV consistency between the two payoff conditions in the interval between [23.18, 30.96].



psychological interpretations of the law of diminishing returns dictate, preferences become more inconsistent when the difference in value is relatively similar (Shevlin et al., 2022). However, the findings were inconsistent (in agreement with Shevlin et al., 2022) with the hypothesis of diminishing value sensitivity. I identified a more complex relationship between relativity and preference consistency. The nature of the relationship between consistency of preference and the relative difference in value was anchored to the absolute difference in value one expects to earn or lose. The study extended the current discussion on the merits of the law of diminishing returns by illustrating a boundary condition for the hypothesis of diminishing value sensitivity. Overall, the results from both studies addressed the limitations of earlier studies and illustrated that the changes in decision strategies were made with respect to the frame of reference of absolute difference (as captured by AED distribution) and not the relative difference in reward (as captured by the payoff conditions).

Summary: Recurring Suboptimality

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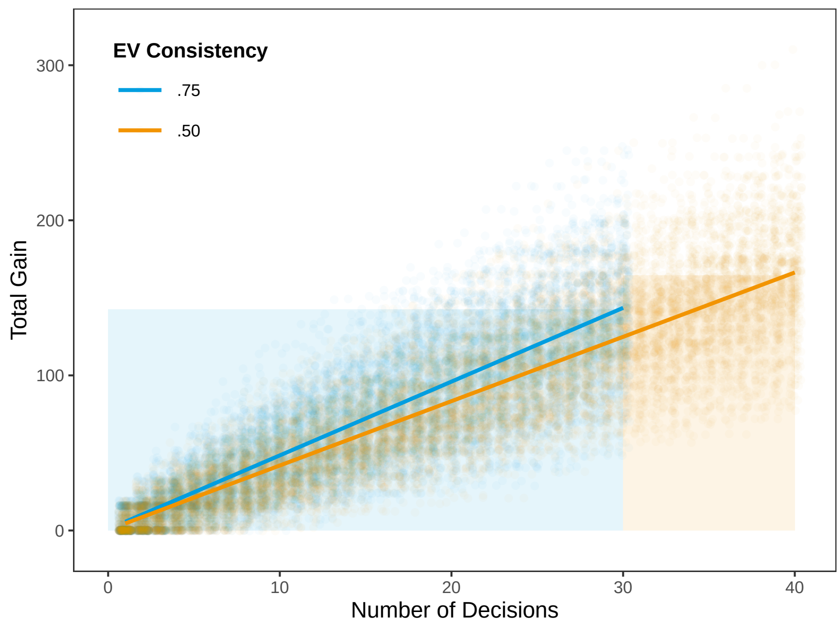
The previous study conclusively demonstrated that the payoff condition alone was insufficient for more numerate decision makers to make adaptive decisions. In the current study, I constructed a more simplified environment where making adaptive decisions is simple yet contrary to existing normative principles of decision-making literature. In other words, I created a task environment where making recurring suboptimal choices is adaptively rational. That is, decision makers who made quick decisions by sacrificing their accuracy, under a resource-constrained environment, would accumulate more rewards than those who spent more time making the best decision. In this task, trying to maximize outcomes through brute force maximization may lead to a worse overall result. In contrast, people who comprehend the structure of the decision task could trade off the decision accuracy and computational cost by making numerous faster suboptimal choices, which would result in better performance in the long run.

I empirically tested this specific pattern of behavior. Specifically, I examined whether decision makers were able to better "see" the structure of the task and adapt their decision strategy in accordance with the constraints associated with it (Dawes, 1979). I tested this hypothesis in one simulation study and three empirical studies.

7.1 Simulation Study

The simulation study aimed to demonstrate that making more suboptimal choices under the current task structure could lead to better overall performance (as measured by a higher total gain) than making decisions following a deliberative normative model. In this study, the normative decision strategy, known as the EV maximization model, was used as a benchmark for accurate behavior under risk to maximize reward. Therefore, choices that did not follow the normative model's prediction were considered suboptimal. To support this claim, simulated responses were generated from the same 10 choice problems used in the empirical studies. As captured by Fig. 7.1, the "blue" decision maker, who followed the more energy-intensive EV maximization strategy 75% of the time, would face only 30 choice problems. On the other hand, the "orange" decision maker, who randomly chose between options with an EV consistency of 50%, would earn more reward by facing 40 choice problems. This suggested that making more suboptimal choices could lead to better overall performance in this context.

Figure 7.1: Total gain as a function of the number of decisions made with different EV consistency. The blue line models decision makers who maximize EV in 75% of 30 decision problems, while the orange line models those who maximize EV in 50% of 40 decision problems (they make random choices). Dots illustrate simulated individual data. Blue and orange areas represent the maximum number of decisions (x-axis) and mean total gain (y-axis) for the two EV-consistency conditions.



Existing literature on speed-accuracy trade-off predicts an inverse relationship between the speed of decisions and the accuracy of those choices (Heitz, 2014; Liesefeld & Janczyk, 2019; Wickelgren, 1977). As a consequence, assuming that making more EV-consistent choices requires more time, decision makers who make faster decisions in a resource-constrained environment by sacrificing their accuracy will encounter more choice problems and earn more rewards in the gain domain than individuals who make more accurate EV-consistent choices.

The purpose of the study was to test the prediction of the simulation study empirically. The first study (i.e., Study 1) served as a control condition where I presented decision problems without any time constraints. I expected that participants, in an unconstrained choice environment, would follow a more deliberative and normative decision strategy to achieve higher total rewards. Next, in the second and third studies (i.e., Study 2 & 3), I would introduce time constraints in the task environment to capture whether participants modulate their decision strategy following the change in resource availability. Under time constraints, I expected some individuals (especially decision makers with higher statistical numeracy due to their superior numeric skills) would better understand the task structure and make recurring suboptimal choices by adapting their exploration strategy to encounter more choice problems and earn higher rewards. In other words, making faster recurring suboptimal (or random in terms of EV maximization) decisions would result in an overall superior performance.

7.2 General Methodology

A diverse sample of N = 1387 volunteers (51.2 % females, M_{age} = 37.37, SD_{age}= 13.91), recruited via the Prolific platform, completed three online studies (N_{Study1} = 350, N_{Study2} = 348, N_{Study3} = 345). Participants were paid £1.88 for a study lasting approximately 15 minutes. Additionally, Each participant received an additional £1 for every 200 points on top of the flat payment they would receive once they completed the task.

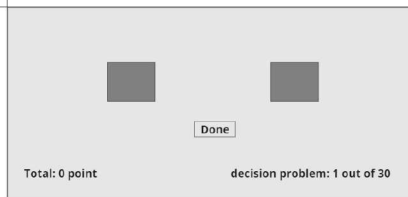
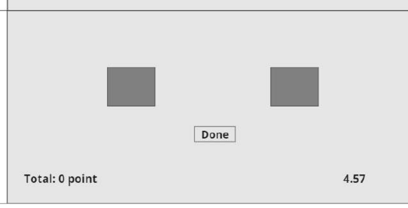
In all studies, I used a Decisions-from-Experience task (DfE) to investigate information search and modulation in decision strategy (Hertwig & Erev, 2009; Hertwig et al., 2004; Wulff et al., 2018). In addition, the numerical skills of decision makers were assessed with BNT (Cokely et al., 2012). The order of tasks (BNT and DfE) was counterbalanced across participants (order effects were not found).

In all three studies, the general procedure was very similar, with the main difference being the time constraints and the sets of choice problems used in the decision task (see Fig. 7.2 for a summary of differences between studies). In Study 1, participants were informed that their task was to make 30 decisions in 30 choice problems without any time constraints. In Study 2 and 3, participants were asked to answer as many problems as possible within a specified time limit of 5 minutes. A timer was displayed during the task to show the elapsed time, counting down from 5 minutes. Additionally, Study 3 presented participants with 120 unique binary choice problems, while Study 1 & 2 had 10 unique problems, consisting of two-outcome gambles in the gain domain. In Study 3, I presented participants with questions measuring fluid intelligence and metacognitive understanding of the choice task structure as well as employed decision strategies.

7.3 Study 1: Unconstrained Task

The objective of Study 1 was to investigate the predictors of superior decision-making in an unconstrained environment. Study 1 served as a control condition for evaluating the effect of time constraints and changes in decision strategies. In this condition, there were no time constraints (i.e., participants would solve exactly 30 problems), allowing participants to explore each choice problem for as long as they needed to decide.

Figure 7.2: Procedural differences between the three studies.

Study	Screenshot	Information about the exact number of choice problems	Information about the elapsed time during the task	Choice Problems	Measures
Study 1		Yes	No	10 unique problems	Choice problems & BNT
Study 2		No	Yes	10 unique problems	Choice problems & BNT
Study 3		No	Yes	120 unique problems	Choice problems, BNT & ICAR

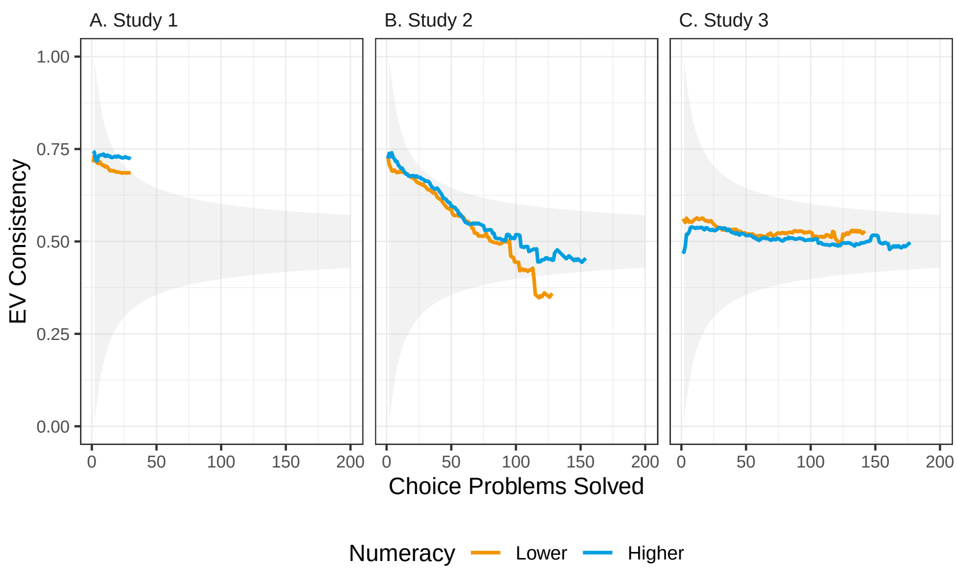
7.3.1 Results & Discussion

Results from Study 1 confirmed that, without environmental constraints, higher total rewards were associated with choices that maximized EV, $r(349) = .266$, $p <.001$. In addition, enhanced task performance correlated with a more thorough search process, characterized by intensive sampling, $r(349) = .326$, $p <.001$, fewer switches between gambles, $r(349) =$ -.121, $p = .023$, and increased time spent exploring, $r(349) = .199$, $p <.001$. Notably, more numerate individuals were found to be better at making more EV-consistent choices, $r(349) = .176$, $p <.001$, by drawing more samples, $r(349) = .249$, $p <.001$, deliberating more, $r(349) = .145$, $p = .007$, and switching less between options, $r(349) =$ -.262, $p <.001$.

To complement the correlation result, I fitted a linear mixed model with numeracy and consecutive choice problems as predictors to examine whether numeracy moderated the relationship between different decision strategies. The result confirmed that more numerate individuals made more EV-consistent decisions, as captured by Fig. 7.3A, while participants with lower numeracy made fewer EV-consistent choices and followed a more random strategy ($b = 0.001$, $t(10150) = 4.033$, $p <.001$).

Overall, Study 1 successfully demonstrated that decision makers who followed normative principles accumulated significantly higher rewards in an unconstrained environment. More importantly, due to their higher numeric proficiency, more numerate individuals could make more normative decisions than less numerate individuals by

Figure 7.3: The mean proportion of choices consistent with the EV maximization strategy (EV Consistency) as a function of the number of choice problems solved and numeracy. The shaded regions denote the 95% CI derived from a binomial distribution for consecutive binary choice problems. The point estimates (depicted as lines) within the 95% CI suggest that the decision strategy is statistically indistinguishable from a random selection strategy.



creating more precise representations of the choice problems. In the following study (i.e., Study 2), I aimed to explore how imposing time constraints on the choice environment would influence decision-making and exploration strategies. Specifically, I investigated whether time constraints would prompt changes in decision and exploration strategy, potentially leading to improved overall performance as a result of adopting recurring suboptimal choices.

7.4 Study 2: Constrained Task

In Study 2, I introduced a time-constrained framework. Unlike in Study 1, participants were unaware of the exact number of choice problems they would encounter. Instead, they were informed that the task would span precisely 5 minutes. I expected that, as predicted by the simulation study, more numerate decision makers, who in an unconstrained environment made more normatively superior decisions by employing more thorough search strategies, would adapt to this time-limited task due to their higher numeric competency and shift to quicker and more random strategy, in terms of EV maximization, to earn a higher total reward.

7.4.1 Results & Discussion

Unlike Study 1, in this current study, total gain was related to the lower proportion of EV-consistent choices, $r(347)$ = -.330, p < .001, the lower number of samples drawn, $r(347)$ = -.630, p < .001, and exploration time, $r(347)$ = -.611, p < .001, but positively associated with the switching ratio, $r(347)$ = .358, p < .001. The empirical results from Study 2 confirmed the simulation result. Participants who made more random choices without strictly following the EV maximization policy were able to save time and make more decisions, ultimately leading to higher overall rewards.

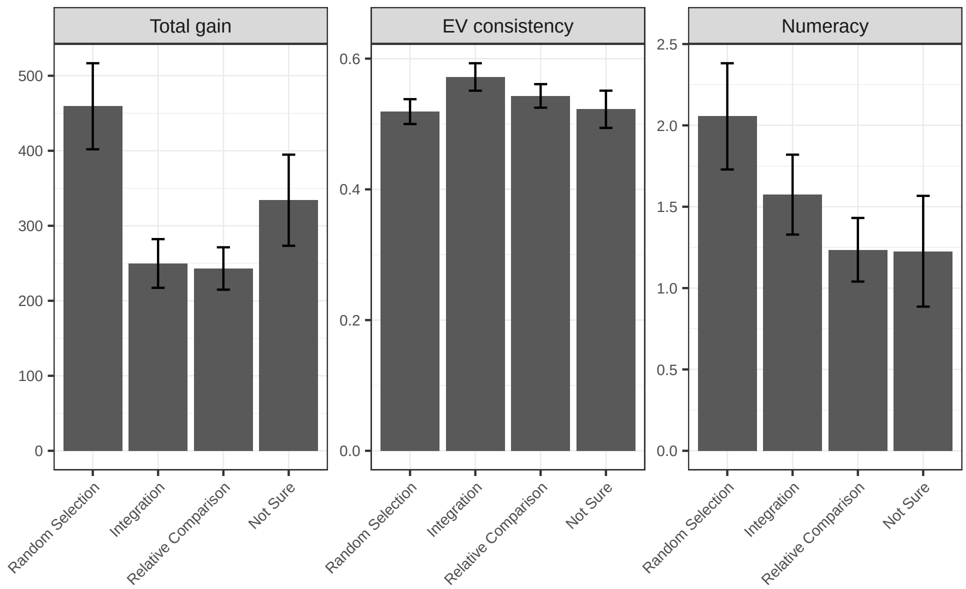
I ran linear regression to examine the relationship between numeracy and suboptimal decision strategy predicting total gain. The result revealed that more numerate individuals performed significantly better in the task, b = 9.43, $t(340)$ = 2.76, p = .006, and the effect remained significant even after the model was adjusted for the exploration measures. This result was further confirmed by decision strategy analysis. In comparison to Study 1, the number of participants classified as followers of the random strategy was higher (41%, 141 participants), and fewer participants were classified as following the EV strategy (48%, 170 participants). This change (between Study 1 and 2) was statistically significant, $\chi^2(1)$ = 7.290, p = .007. Participants with higher numeracy made random choices throughout the study (as captured by Fig. 7.3B), while participants with lower numeracy made more choices that were not predicted by the EV maximization strategy. In other words, participants, especially more numerate individuals, made more random choices without strictly following the EV maximization policy to save time and make more decisions, which ultimately led to higher overall rewards.

7.5 Study 3: Unique Problems

In Study 2, the experiment was limited to only 10 unique choice problems. Consequently, participants who made more choices had the chance to solve the same problems multiple times, potentially relying on memory for their decisions. Additionally, although I observed various search strategies that participants used to achieve higher rewards, it was still unclear if those who performed better in the task developed a metacognitive understanding of the choice problems and the environment's structure. Addressing these limitations was the primary objective of Study 3.

I used a new set of 120 unique binary choice problems consisting of two-outcome gambles in the gain domain to differentiate between a compensatory strategy, as captured by expectation models like EV or CPT (Tversky & Kahneman, 1992), and a heuristic strategy, as represented by the PH model (Brandstätter et al., 2006). Besides the choice problem and BNT scale, in Study 3, participants were additionally asked to solve ICAR and questions regarding their metacognitive understanding after the DfE task but before informing participants of their total gain.

Figure 7.4: Mean total gain, EV consistency, and numeracy score as a function of the reported strategy classification in Study 3. Error bars represent the 95% confidence intervals.



7.5.1 Results & Discussion

Similar to Study 2 and in contrast to Study 1, total gain was related to the lower proportion of EV-consistent choices, $r(344)$ = -.160, p = .001, the lower number of samples drawn, $r(344)$ = -.688, $p <$ .001, and exploration time, $r(344)$ = -.676, $p <$ .001, but positively associated with the switching ratio, $r(344)$ = .305, $p <$ .001. Similarly to Study 2, the regression analysis indicated that within a fixed time frame, participants with higher numeracy performed better in the task, b = 13.07, $t(338)$ = 2.022, p = .044, after adjusting the model for exploration measures. In other words, I successfully replicated the main findings from Study 2, albeit with a different set of 120 unique choice problems. In addition, participants who made more recurring suboptimal choices, spent less time on individual problems, encountered more problems, and earned higher rewards as predicted by the simulation results.

The number of participants classified as followers of the random strategy was higher (69%, 238 participants), and fewer participants were classified as following the EV strategy (22%, 76 participants). This change (between Study 1 and 3) was statistically significant, $\chi^2(1)$ = 108.407, $p <$ .001. Furthermore, more numerate participants, consistent with results from study 2, were less likely to make more EV-consistent choices, b = -0.037, $t(349.4)$ = -2.465, p = .014, and numeracy moderated the effect of the number of choice problems solved on EV consistency, b = 0.001, $t(14980)$ = 12.285, $p <$ .001.

Finally, the result from the open-ended questions, as captured by Fig. 7.4 about the reported strategies used by participants, further corroborated the results derived from the model's prediction. The analysis of the written report was done by engaging four independent judges to categorize each participant's response into one of the four predefined categories (i.e., Random Selection, Integration, Relative Comparison, Not Sure). Consistent with the analysis of Study 2 & Study 3, the Random Selection strategy achieved significantly higher rewards compared to those who chose other strategies (all p-values $< .001$). Furthermore, the mean BNT score was significantly higher among participants who made random choices compared to all other strategies (all post hoc p-values $< .05$). Put simply, the result from open-ended questions successfully demonstrated that participants recognized the effectiveness of the Random Selection strategy in this task environment and earned more rewards. The average numeracy level was also highest among those who adopted this random strategy.

7.6 Conclusion

I found, consistent with the existing theories of decision making, that normative choices typically yield significantly higher rewards than repeated suboptimal decisions in an unconstrained environment. However, confirming the primary hypothesis and prediction from the simulation study, I found that decision makers who consistently made suboptimal random choices under time constraints accrued higher total rewards than those who opted for deliberate, energy-intensive normative decisions. Importantly, my main contribution is to conclusively demonstrate that individuals with higher numeracy adaptively modified their exploration strategies in response to the alterations in the task environment. Specifically, they engaged in more dynamic and flexible information sampling and employed smart search strategies, predominantly focusing on reducing uncertainty and uncovering unobserved outcomes. This adaptability facilitated the development of a metacognitive understanding of both the structure of the task and the choice environment, leading to the predominant use of random choices to accumulate higher total rewards.

These results extended the current discourse on descriptive theories that attempted to explain decision making. Rational decision theories and even common sense suggest that optimal decision making results from selecting an option for which the sum of the outcomes weighted by the probability of their occurrence is the highest (Baron, 2008). Astonishingly, I found the opposite effect—individuals who made more suboptimal choices objectively outperformed those who were more likely to make choices that maximized EV. In light of contemporary psychological models, such behavior cannot be regarded as rational (Stanovich, 1999).

I argue that superior or optimal decision making cannot be fully realized without considering the constraints associated with task characteristics (e.g., time limits, uncertainty, opportunity cost) and human reasoning (e.g., resource limitation, cognitive thresholds; Bhui et al., 2021; Lieder & Griffiths, 2020).

Discussion

Existing literature from the field of expert decision making robustly demonstrated that skilled decision makers were able to better understand the context (i.e., environmental and cognitive constraints) in which they operate and were able to modulate their decision strategies to suit specific scenarios (Ericsson & Pool, 2016). In the thesis, my goal was to investigate this result for decision makers proficient (i.e., skilled) in understanding numerical and statistical information. If expertise is the reason skilled decision makers were able to make more adaptive decisions, more numerate individuals, due to their proficiency in number processing, in principle, should be able to better understand the statistical regularities of the numerical environment to make better decisions than less numerate individuals. In the thesis, I examined this line of argument and investigated the relationship between numeracy and adaptive decision making by creating an environment where adaptive decisions are orthogonal to normative decisions. The goal was to establish a relationship between the psychological plausibility of individuals' behavior and its ecological effectiveness. Through the thesis, I explored the idea that individual choices could violate normative principles, yet they can be reasonable. Due to the contrary nature of it, this construct enabled me to effectively identify when more skilled individuals (i.e., more numerate individuals) were making more adaptive choices (i.e., suboptimal decisions) than less numerate individuals. The results from the six empirical studies and one simulation study support this line of argument.

The first two articles examined the effect of context in adaptive decision making. The study published in the first article (see chapter 5 for a summary) replicated the finding from Traczyk, Sobkow, et al. (2018) where authors found that more numerate individuals were able to modulate their decision strategy by recognizing the relative difference in the payoff. However, the subsequence extension of the study demonstrated a different result (see Chapter 6 for a summary). When multiple factors (i.e., lack of trade-off between strategies, asymmetry in AED distribution) were controlled, more numerate individuals, compared to less numerate individuals, were better at maximizing expected reward following the absolute difference in value between options and not the relative difference in value embodied by two payoff conditions. The Johnson-Neyman interval of insignificance indicated that decision makers, irrespective of their level of numerical understanding, used AED distribution as a reference to make their decisions, consistent with the prediction from Adaptation-Level Theory (ALT; Helson, 1964). The ALT provides a useful structure for understanding the stimulus frame of reference, enjoying substantial support within perceptual and psychophysical research fields. The theory plays a critical role in explaining a variety of phenomena, including constancy, contrast, and sensory adaptation, across varied perceptual domains such as vision, hearing, smell, and taste (Bevan et al., 1962; Helson, 1947; Hulshoff Pol et al., 1998). I found that the adaptation level exists in the cognitive domain as well, in the form of adaptation to value distribution. The ALT suggests that people make judgments based on their adaptation level, influenced by all past and present stimuli. In simple terms, ALT proposes that individuals respond to current stimuli using a frame of reference that is shaped by all previous stimuli (Helson, 1948). This adaptation level can be defined as a region on the stimulus scale that produces indifferent responses. Results from the Johnson-Neyman interval of insignificance in both studies, as captured by Fig. 6.3, validated the notion of adaptation level and the region of indifferent responses in the cognitive domain.

The findings also highlighted the importance of context needed for numerically proficient individuals to make adaptive decisions. Contexts such as lack of trade-off between strategies, asymmetry in AED distribution, and varying difficulty levels provided valuable information needed for highly numerate individuals to make more adaptive choices (make normative choices in important problems and suboptimal heuristics decisions in trivial problems). However, when I controlled such factors to let just the EV ratio vary between two payoff conditions, participants did not adaptively modulate their decision strategy by following the changes in payoff conditions. Instead, they started maximizing EV following AED between options regardless of their numeracy levels. Developmental literature indicates that age has a moderating effect on the ability to understand contextual information. As we get older, our life experiences help us understand tones, facial expressions, and body language, allowing us to interpret the context of sentences, from sarcasm to passive-aggressive anger (Wlotko & Federmeier, 2012). Similarly, the current result indicated that more numerate individuals, due to their superior numeric skills, were able to take in more statistical information to create an accurate and nuanced representation of the choice problem and make better adaptive decisions in that task context while saving time and effort.

In the third publication, instead of providing contextual information in the form of numerical difference (i.e., a significant contrast in relative difference in reward between two payoff conditions), I used a more straightforward environmental limitation (i.e., time constraints; see Chapter 7 for a summary). Participants, especially more numerically proficient participants, were able to discern the statistical structure of the time-constrained task environment (Dawes, 1979) and consequently adopted a strategy of recurring irrationality to achieve higher overall rewards compared to their less skilled counterparts. In the unconstrained task structure, individuals with higher numeracy skills spent more time on each problem, made fewer switches between options, and sampled more from each option to obtain a more accurate verbatim representation of the problem in order to find normatively superior alternatives than less numerate individuals (Broniatowski & Reyna, 2018). However, in the time-constrained environment, individuals with higher numeracy significantly changed their strategy through a more dynamic sampling (i.e., a greater number of samples and switches between gambles within a shorter time frame) and the adaptive reliance on search strategies, compared to their less numerate counterparts. This effect could be explained by theoretical models suggesting that skilled decision making is driven by representative understanding (Cokely et al., 2018) and gist representation (Reyna & Brust-Renck, 2020) rather than rational optimization. Put simply, while in an unconstrained environment, normative choices typically yielded significantly higher rewards than repeated suboptimal decisions, I found that individuals (especially more numerate individuals) recognized, captured by participants verbal report in the metacognitive questionnaire, that the efficacy of a rapid, random choice strategy in a resource-constrained environment resulted in significantly higher rewards (see Fig 7.4). This result repeatedly confirmed the primary hypothesis of the thesis. That is, the adaptive qualities of highly skilled decision makers could be generalized to those who are highly proficient in dealing with numbers (i.e., more numerate individuals) when dealing with numeric problems.

The current result also confirmed the description of resource rational decision making (Lieder & Griffiths, 2020). However, I found numeracy moderating the resource rational behavior. In other words, more numerate individuals were more efficient and accurate in their search strategies, time management, and metacognitive understanding of the task structure, which led them to make better decisions (characterized by repeated quick suboptimal decisions in the constrained environment) than less numerate individuals. More numerate individuals, compared to less numerate individuals, made more random choices in the time-constrained environment without strictly following the EV maximization policy to save time and make more decisions, ultimately leading to higher overall rewards. This result not only captures resource rational behavior but also highlights that choices could violate normative principles, yet they can outperform normative predictions. This is my primary contribution to the field. I demonstrated why considering rationality in light of strict principles may not always capture the quality of overall choices. Unlike the studies in the field of heuristics and biases (Tversky & Kahneman, 1974), ecological rationality (Gigerenzer & Selten, 2002), or the accuracy-effort trade-off framework (Payne et al., 1993), I demonstrated that making suboptimal choices not only approximates the standard normative choices but can also outperform them. I have found that decision makers who followed a normatively superior strategy earned significantly less overall rewards than decision makers who made more random choices under time constraints due to the time associated with following computationally complex processes.

In my thesis, I focused on choice problems within the gain domain due to ethical considerations. As a result, the conclusions drawn from the studies may not directly apply to the loss domain or other non-lottery-based tasks. Monetary lotteries allowed me to examine the foundations of decision making in a controlled laboratory setting. I believe that this approach provided a promising starting point to test whether there is a relationship between numeracy and adaptive decision making in real life or in scenarios involving losses. However, future studies should investigate this line of argument empirically. I am of the opinion that the current findings of recurring irrationality and the influence of context should prompt decision researchers to reconsider the criteria for good decisions and should motivate the redesign of interventions aimed at promoting good choices (Hertwig & Grüne-Yanoff, 2017). Individuals and policymakers should recognize that, under certain conditions, habitual suboptimal choices can lead to improved decision-making. Efforts to train individuals in the rational use of available cognitive resources, particularly in situations with environmental resource constraints, could enhance decision-making abilities in complex and dynamic settings. Interventions with such qualities could be especially beneficial for individuals with lower numerical abilities.

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A List of Scientific Publications Included in the Dissertation

1. Mondal, S. (2021). Sensitivity of numerate individuals to large asymmetry in outcomes: A registered replication of Traczyk et al.(2018). *Decyzje*, (35), 5-26. https://doi.org/10.7206/DEC.1733-0092.150a

2. Mondal, S., & Traczyk, J. (2023). Conditionality of adaptiveness: Investigating the relationship between numeracy and adaptive behavior. *Journal of Economic Psychology*, *96*, 102611. https://doi.org/10.1016/j.joep.2023.102611

3. Mondal, S., Lenda, D., & Traczyk, J. (2024). Recurring suboptimal choices result in superior decision making. *Decision*. Advance online publication. https://doi.org/10.1037/dec0000240

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10.1 First Published Manuscript

SENSITIVITY OF NUMERATE INDIVIDUALS TO LARGE ASYMMETRY IN OUTCOMES: A REGISTERED REPLICATION OF TRACZYK ET AL. (2018)

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Abstract: *The main aim of this study is to replicate the effect shown by Traczyk et al. (2018), where individuals with higher statistical numeracy, compared to individuals with lower statistical numeracy, employed a more effortful choice strategy when outcomes were meaningful. I hypothesize that participants with higher numeracy will be more likely to make choices predicted by Cumulative Prospect Theory and Expected Value theory (CPT/EV) in high-payoff problems than in low-payoff problems. Data collection was done online by appointing 73 participants. Participants' preference, fluid intelligence, objective and subjective numeracy were measured using thirteen high and eleven low payoff choice problems, International Cognitive Ability Resource (ICAR), Berlin Numeracy Test (BNT), and Subjective Numeracy Scale (SNS), respectively. All the measures mentioned above were presented randomly. Results showed that all participants, in high-payoff condition, on average maximized EV; however, participants with high BNT scores were more likely to make choices consistent with CPT/EV predictions than individuals with low BNT scores. Furthermore, compared to less numerate participants, highly numerate participants were less likely to make choices consistent with CPT/EV predictions in low-payoff condition. Highly numerate individuals adjusted their choice strategy by modulating their response time, indicating their discernible sensitivity towards large asymmetry in payoff. In conclusion, the effect shown by Traczyk et al. (2018) was successfully replicated.*

Key words: *Numeracy; Strategy selection; Risky choice; Priority heuristic; EV maximization strategy; Cumulative prospect theory.*

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WRAŻLIWOŚĆ NA ASYMETRIĘ W WYPŁATACH WŚRÓD OSÓB Z WYSOKIM POZIOMEM ZDOLNOŚCI NUMERYCZNYCH. PREREJESTROWANA REPLIKACJA BADANIA TRACZYKA I IN. (2018)

Streszczenie: *Głównym celem tego badania była próba zreplikowania efektu wykazanego przez Traczyka i in. (2018), zgodnie z którym osoby z wyższym poziomem statystycznych zdolności numerycznych, w porównaniu do osób z niższym poziomem statystycznych zdolności numerycznych, angażują wymagające poznawczo strategie decyzyjne, gdy potencjalne konsekwencje wyboru są znaczące. Postawiłem hipotezę, że osoby z wysokim poziomem statystycznych zdolności numerycznych będą częściej dokonywały wyborów przewidywanych przez skumulowaną teorię perspektywy i model wartości oczekiwanej (CPT/EV) w problemach decyzyjnych z wysokimi wypłatami (tj. znaczącymi konsekwencjami) niż w problemach decyzyjnych z niskimi wypłatami. W badaniu online 73 ochotników podejmowało decyzje w 13 problemach z wysokimi wypłatami oraz w 11 problemach z niskimi wypłatami. Badani rozwiązywali testy mierzące inteligencję płynną, statystyczne zdolności numeryczne oraz subiektywne zdolności numeryczne. Wszystkie miary były prezentowane w losowej kolejności. Wyniki pokazały, że w warunkach wysokiej wypłaty osoby badane dokonywały wyborów maksymalizujących wartość oczekiwaną. Osoby z wysokimi wynikami w teście mierzącym statystyczne zdolności numeryczne częściej dokonywały jednak wyborów zgodnych z przewidywaniami CPT/EV niż osoby z niskimi wynikami w tym teście. Ponadto osoby z wysokim poziomem statystycznych zdolności numerycznych były mniej skłonne do dokonywania wyborów zgodnych z przewidywaniami CPT/EV w warunku niskiej wypłaty. Osoby te dostosowały swoją strategię wyboru do problemu decyzyjnego poprzez zarządzanie czasem przeznaczanym na podjęcie decyzji, co wskazuje, że mogą one charakteryzować się większą wrażliwością na asymetrię w wypłatach. Podsumowując, efekt opisany w badaniu Traczyka i in. (2018) został pomyślnie zreplikowany.*

Słowa kluczowe: *zdolności numeryczne; strategie decyzyjne; ryzykowny wybór; heurystyka pierwszeństwa; strategia maksymalizacji wartości oczekiwanej; skumulowana teoria perspektywy.*

Every moment of our life is bombarded with information condensed in a statistical shell (Rothman et al., 2006). In order to make informed decisions, from giving tips to buying dogecoin, one on a daily basis, needs to comprehend and calculate various kinds of statistical information. Out of numerous decisions we make every day, seldom do we come across decisions that can have a momentous impact (Cirillo & Taleb, 2016; Taleb, 2020). The quality of our judgments in those crucial moments is highly dependent on each individual's level of expertise. For example, a statistician would be much less hopeful (assuming the person has done the math) about their prospect of winning a lottery compared to a person who lacks knowledge in the field and probably would continuously buy lottery tickets year after year with the hope of being a millionaire one day. Hence, expertise modulates human preferences as well as expectations concerning those choices (Reyna, Nelson, Han, & Dieckmann, 2009).

Many studies have examined the effect of individual differences (i.e., numeracy, intelligence, personality traits, and so on) on human preferences (Becker, Deckers, Dohmen, Falk, & Kosse, 2012; Sobkow, Garrido, & Garcia-Retamero, 2020; Traczyk & Fulawka, 2016). For example, patients with low numeracy often do not have accurate perception of benefits and risks associated with unproven medical treatments and interventions. Moreover, less numerate patients fail to accurately consider the reported prevalence rate of diseases, which skews their perception of personal risk of suffering several diseases compared to individuals with high numeracy (Davids, Schapira, McAuliffe, & Nattinger, 2004; Gurmankin, Baron, & Armstrong, 2004).

A significant amount of evidence has also been accumulated regarding the role of numeracy in the context of financial decision-making in the last decade (Jasper, Bhattacharya, & Corser, 2017; Lusardi, 2012; Sobkow, Garrido, & Garcia-Retamero, 2020). In economic theory, optimal behavior under risk and uncertainty is interpreted by variants of expected value or expected utility models. These theories were proposed as a normative rational choice theory, where a rational agent should select an action that is expected to maximize its outcome (for an introduction, see Małecka, 2020). Although, when the normative theory was put to the test, it revealed that humans did not follow normative standards all the time. Therefore, a positive theory of behavior was proposed (i.e., prospect theory and later the cumulative representation of prospect theory) (Kahneman & Tversky, 1979; Thaler, 1980). It is a modification of expected utility theory while keeping the framework of expected utility theory. Prospect theory used another set of psychological variables (i.e., reference point and non-linear weighting function) to address the discrepancies between normative models and human preferences but adhering to the assumption that human preference can be successfully modeled by weighting and summing operations inherited from expected value calculation (Hands, 2015; Tversky & Kahneman, 1992).

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Recent results point out that objectively numerate individuals are more sensitive to changes in expected value compared to less numerate individuals (Jasper, Bhattacharya, Levin, Jones, & Bossard, 2013). Furthermore, highly numerate individuals are also more consistent in their preferences regardless of how information is presented compared to individuals with low statistical knowledge. Numerate participants consistently choose riskier options in both decisions-from-description and decisions-from-experience task, providing evidence for their consistency (Ashby, 2017). Interestingly, Cokely and Kelley (2009) showed that despite the positive relationship between numeracy and choices maximizing Expected Value (EV; Bernoulli, 1954; Russell & Norvig, 2002), protocol analyses revealed that individuals with high numeracy did not commonly use EV calculations to arrive at those choices. Instead, retrospective verbalization revealed that participants used elaborative heuristic search processes to make their decisions. Hence, the authors concluded that superior decisions could also be made with simple heuristic processes instead of energy-intensive weighting and summing operations (Gigerenzer, 2007; Gigerenzer & Goldstein, 1996). In addition, one can also interpolate that numerate individuals have a wider repertoire of decision strategies given that numerate individuals' decisions resemble EV maximization strategy even though they are implementing heuristic processes. Put differently, numerate individuals are equipped with the toolkit necessary to use both an energy-intensive EV calculation strategy and can also rely on simple heuristic processes.

In order to test the aforementioned conjecture, Traczyk et al. (2018) conducted a study examining whether people with high objective numeracy modulate their strategy to the consequence of the decision or whether they simply make normatively superior decisions regardless of the magnitude of the outcome. The authors observed that individuals with higher objective numeracy maximized EV and made choices consistent with the predictions of Cumulative Prospect Theory (CPT; Tversky & Kahneman, 1992) when the EV ratio difference between gambles were high. However, in problems where the EV ratio between gambles were low and the potential outcomes were comparable, highly numerate participants adapted their strategy and made choices consistent with the predictions of the Priority Heuristic (PH; Brandstätter, Gigerenzer, & Hertwig, 2006) and, on average, did not maximize EV or made decisions predicted by CPT compered to less numerate participants.

The main aim of the current study is to replicate the effect studied by Traczyk et al. (2018) where people with higher statistical numeracy, in comparison to people with lower statistical numeracy, strategically employ a more effortful choice strategy to make adaptive choices when the choice problem is meaningful. That is, I seek to replicate the effect where participants with high statistical numeracy (i.e., the ability to understand and process numerical and statistical information) will be more likely to make choices consistent with the prediction made by CPT/EV, compared to participants with low statistical numeracy, in high-payoff choice problems but not in low-payoff choice problems.

Method

The current study is a pre-registered close replication study. Complete pre-registration, experimental procedure, sample size estimation (R scripts), data used for analysis, complete analysis (R markdown file), and supplementary materials have been posted on the Open Science Framework (https://osf.io/cje9b/).

Procedure and materials

The current study is investigating the relationship between statistical numeracy and choices under asymmetric payoff conditions. Participants' fluid intelligence, objective numeracy, and subjective numeracy was measured using the International Cognitive Ability Resource (ICAR; Condon & Revelle, 2014), Berlin Numeracy Test (BNT; Cokely, Galesic, Schulz, Ghazal, & Garcia-Retamero, 2012), and Subjective Numeracy Scale (SNS; Fagerlin et al., 2007), respectively. Participants also responded to, randomly presented, thirteen high-payoff (outcome difference between two gambles is high) and eleven low-payoff (outcome difference between two gambles is low) choices in binary two-outcome gambles framed as gains.

Participants were instructed to complete the procedure individually during one session. They were further asked not to use a calculator and turn off any devices that might cause inattentiveness during the session. Tasks were designed in Inquisit Web (2016) software and ran in the Prolific platform. Texts were displayed in black font on a light gray background. During one session, after the demographic questionnaire, participants were asked to answer BNT, SNS, ICAR, and choice problems all presented in a random order.[1] The entire procedure was presented in English and took 25 minutes on average (although there were no time constraints) to complete.

Objective statistical numeracy. In the article, objective statistical numeracy is defined as a metric to differentiate individuals proficient in probabilistic and statistical computations (Cokely et al., 2012; Lipkus, Samsa, & Rimer, 2001; Schwartz, Woloshin, Black, & Welch, 1997). The Berlin Numeracy Test was used to measure objective statistical numeracy, risk literacy, and comprehension of probabilistic concepts. A computerized version of the Berlin Numeracy Test was used

[1] Presentation of each measure were done based on a sequence generated randomly.

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in the current study consisting of four items presented to participants in a predefined order. Possible scores ranged from 0 to 4 points, with higher scores indicating higher objective statistical numeracy.

Subjective numeracy. Subjective numeracy measures an individual's perception of their numeracy (Fagerlin et al., 2007). In the study, subjective numeracy was measured using an 8-item self-assessment Subjective Numeracy Scale that includes two sub-scales referring to perceived numerical abilities (e.g., "How good are you at calculating a 15% tip?") and preference for numerical and statistical information in daily life (e.g., "How often do you find numerical information to be useful?"). Participants were instructed to choose options that best represent their beliefs about themselves. Possible scores ranged from 0 to 48 points, with higher scores indicating higher subjective numeracy.

Fluid intelligence. Fluid intelligence can be defined by an individual's ability to use reasoning to solve abstract problems without or minimally using prior learning (McGrew, 2021). Four matrix reasoning items from ICAR were used to measure fluid intelligence (Condon & Revelle, 2014). Reasoning problems were presented in the form of three-by-three matrices of elements with one missing element. Participants were instructed to identify the rule underlying the matrix and select one of the six response elements that satisfied the rule. Possible scores ranged from 0 to 4 points, with higher scores indicating higher fluid intelligence.

Choice problems. Being a replication study, almost the same 24 (except one)[2] choice problems were used from the original study. Each choice problem was classified either as a low or high payoff problem based on the EV ratio between gambles. When EV ratio between gambles are low (i.e., 1.5-1.6), choices are considered low-payoff choice problems because playing them repeatedly, on average, would lead to relatively small differences in payoffs irrespective of the chosen gambles. Hence, it is assumed that low-payoff choice problems are trivial because much less consequence is attached when participants are choosing between options. Notwithstanding, when EV ratios between gambles are high (i.e., 5.56-5.87), choices are considered high-payoff choice problems because the EV of each gamble differs significantly. Therefore, it is assumed that high-payoff choice problems are meaningful because choosing any gamble with the higher EV, on average, will lead to much higher payoffs.

These choice problems were explicitly selected to distinguish between the strategy predicted by heuristic strategy (PH) and weighting and summing operation (CPT/

[2] One choice problem differed from the original study because of inappropriate translation of research materials from Polish to English.

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EV). Put differently, choice problems were designed specifically to distinguished between weighting and summing operations embodied by compensatory expectation models (i.e., CPT/EV), and heuristics non-compensatory simple processes relying on trade-offs (i.e., PH). For example,

Gamble A:	$5.40 with 29%;	$0 with 71%
Gamble B:	$9.70 with 17%;	$0 with 83%

PH predicts that a decision-maker will choose Gamble A because the difference in minimum gain in probabilities is larger than 10% of the probability scale (i.e., 0.71 vs. 0.83). In contrast, CPT with standard parameters from Tversky and Kahneman (1992) predicts that a decision-maker will choose Gamble B because of its greater CPT value (i.e., 1.38 vs. 1.78). Under the current experimental procedure, CPT predictions are to be the same as EV maximization strategy.[3] Therefore, whatever participants decide, it will match with either PH theory's prediction or will resemble EV maximization strategy/CPT theory's prediction (for more elaboration, see Pachur, Hertwig, Gigerenzer, & Brandstätter, 2013). Regardless of the participant's choice, it does not imply that participants conform to either theory. Instead, it was intended to examine and track changes (if any) in strategy (compensatory to non-compensatory) corresponding to changes in the payoff structure.

Differences between the original and replication study. Unlike the original study (Traczyk et al., 2018), the current study is not using the Need for Cognition Scale (NCS; Cacioppo & Petty, 1982) and Raven's Advanced Progressive Matrices (RAPM; John & Raven, 2003). Instead, the current study uses International Cognitive Ability Resource (ICAR; Condon & Revelle, 2014) as a replacement for RAPM. Second, the mode of instruction between the original and the current replication study is different. The original study was in Polish, but the current replication study is in English. As a consequence, participants in the original study belonged mostly from Poland, but anyone proficient in English can partake in the replication study. This might include a more heterogeneous sample, which may have an influence on the effect. Notwithstanding, to mitigate the effect of instruction difference, I have used the help of Google Translator and bilinguals (proficient with both Polish and English) to make the translation as accurate to the original as feasible without distorting the meaning. Second, to control the potential effect of ICAR introduction, I have randomized presentations of each block to counterbalance the effect.

[3] CPT prediction is aligned with EV maximization strategy for the current set of choice problems. All the gambles CPT predicts also have higher EV (i.e., EV of 1.57 vs. 1.65 for Gamble A vs. Gamble B). Therefore, when participants choose any gamble predicted by CPT, it means choosing a gamble with a higher EV value.

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Participants

The sample size was determined by simulation using the data collected in the original study. I used the Generalized Linear Mixed Model framework to estimate sample size to obtain 95% statistical power.

Sample size estimation model.

$$log\left[\frac{p(choice=1)}{1-p(choice=1)}\right] = \beta_0 + \beta_1(BNT) + \beta_2(SNS) + \beta_3(NCS) + \beta_4(RAPM) + \beta_5(Payoff) + \beta_6(Payoff:BNT) + \beta_7(Payoff:SNS) + subject_{0s} + e_{si} \quad (1)$$

Where,

$$subject_{0s} \sim N(0, \tau_{00}^2),$$
$$e_{si} \sim N(0, \sigma^2).$$

Using the aforementioned model, I calculated the effect size for the interaction term between statistical numeracy (measured by the Berlin Numeracy Test) and payoff (high vs. low). Considering the effect size estimated in the original study (R^2 = .012, d = 0.442; Brysbaert & Stevens, 2018), simulation with 1000 random data points suggest that 75 participants would be sufficient to obtain a significant (p < .05) interaction effect between BNT and payoff with 95% statistical power. (Arnold, Hogan, Colford, & Hubbard, 2011; Johnson, Barry, Ferguson, & Müller, 2015).

Out of seventy-five participants, only two did not finish the entire study; hence their data is eliminated (in accordance with the disclosure made in the pre-registration form). Seventy-three adult volunteers (*age range*: 19-57 years; *mean* = 27 years) participated in an online study for a half-hourly compensation of £4.00 GBP (equivalent to approximately $5.5 USD). Participants were recruited via the Prolific platform, where they were explicitly told that the current study only examines their cognitive abilities, and compensation was by no means based on their performance in the study. Participation in the study was voluntary, and participants could quit the study at any time without any consequences. Participants gave informed consent before starting the study. The departmental ethics committee of SWPS University of Social Sciences and Humanities approved the study protocol.

Results

In order to examine how measures of individual differences[4] interact with varied payoff structure, Pearson's correlation were calculated and presented in Figure 1. ICAR (a measure of fluid intelligence) is significantly correlated with both BNT (a measure of objective numeracy) and SNS (a measure of subjective numeracy), $r = .32$ ($p = .006$), and $r = .30$ ($p = .011$), respectively; however, BNT and SNS themselves have a negligible correlation of $r = .04$ ($p = .744$) unlike previous studies. Due to this unusual result, Cronbach's α was calculated for both SNS ($\alpha = 0.78$) and BNT ($\alpha = 0.62$). In high-payoff choice problems (EV ratio is relatively high), both SNS and ICAR have a positive correlation of $r = .34$ ($p = .003$), and $r = .32$ ($p = .005$), with choices predicted by CPT/EV, respectively. However, higher scores in BNT are negatively correlated with CPT/EV consistent choices in low-payoff conditions (EV

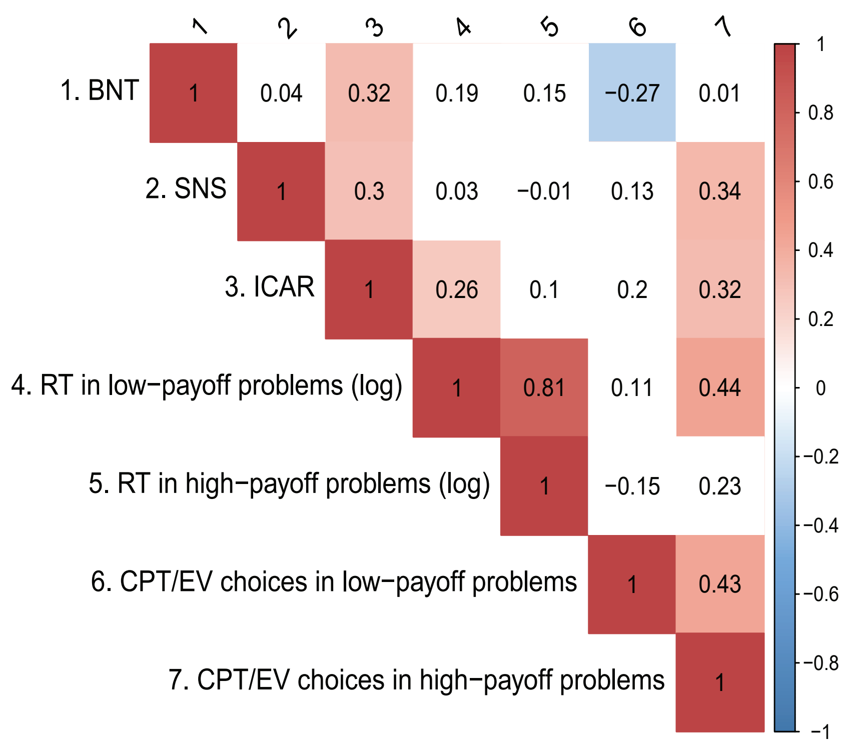

***Figure 1*. Pearson's zero-order correlation coefficient matrix illustrating the relationships between measures used in the study. Significant correlations are marked with color. Here, BNT – Berlin Numeracy Test; SNS – Subjective Numeracy Scale; ICAR – International Cognitive Ability Resource; CPT/EV choices and RT in low and high payoff problems refer to response time and choices consistent with expected value predictions.**

[4] The descriptive table for individual difference measures are in the supplementary material section (https://osf.io/65xdq/).

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ratio is relatively low) with a coefficient value of $r = -.27$ ($p = .021$). Lastly, there is a significant correlation of $r = .26$ ($p = .027$) between ICAR and Response Time (RT; the time participants spend in each trial before making a decision.) in low-payoff choice problems, whereas there is negligible correlation of $r = .1$ ($p = .4$) between ICAR and RT in high-payoff choice problems.

Next, Mann-Whitney test was conducted to illustrate the difference in choice strategy between participants with varying levels of numeracy. There is a significant difference in choice strategy ($W = 72242.5$, $p = 0.003$) between participants with high BNT scores compared to participants with low BNT scores in low-payoff condition with an effect size of -0.09. Similarly, participants with high BNT scores also followed significantly different choice strategy than participants with low BNT scores in high-payoff condition ($W = 118852.5$, $p = 0.006$) with an effect size of 0.07.

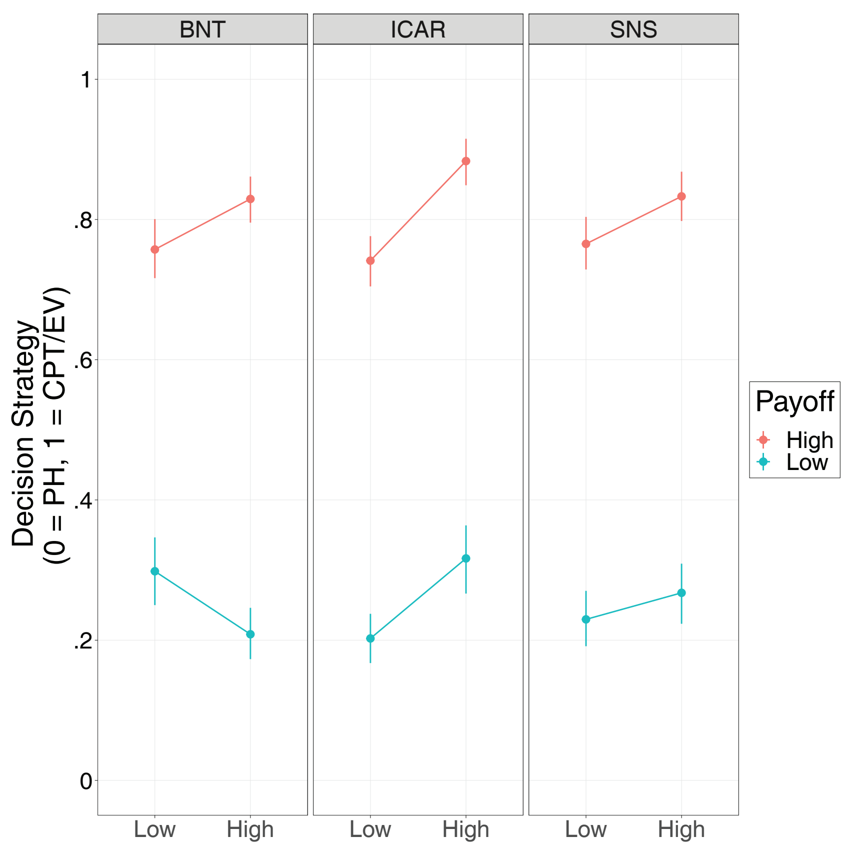

Figure 2. **Decision strategy as a function of BNT, ICAR, and SNS scores. Changes in decision strategy under varied payoff condition illustrated using different colours. Here, BNT – Berlin Numeracy Test; SNS – Subjective Numeracy Scale; ICAR – International Cognitive Ability Resource; 0 = PH refers to choices consistent with Priority Heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.**

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Figure 2 affirms the aforementoned results and further communicates how subjective numeracy, fluid intelligence, and objective numeracy predict different aspects of human decision-making. These differences are especially apparent in low-payoff conditions, where participants with higher BNT scores made decisions more consistent with the strategy predicted by PH as opposed to participants with higher SNS and ICAR scores.

This contrasting strategy selection is what motivated me to use a multivariate analysis technique such as Canonical Correlation Analysis (CCA) to test the relationships between variables (i.e., BNT, SNS, ICAR, CPT/EV consistent choices, and payoff) without committing, or minimizing the probability of committing, a Type I error (Sherry & Henson, 2005). I performed bivariate correlation (Pearson r) between Canonical Variate 1 (CV1) and Canonical Variate 2 (CV2). CV1 is a synthetic predictor variable consist of linear combination of BNT, SNS, & ICAR. In contrary, CV2 is a synthetic criterion consist of linear combination of CPT/EV consistent choices, and payoff sensitivity.

Table 1
Loadings on CV1, & CV2

	CV 1	CV 2
BNT	-0.31	0.847
SNS	0.65	0.551
ICAR	0.705	0.311
	CV 1	**CV 2**
Choice	0.817	-0.577
Payoff	-0.005	0.999

Table 1 shows the weights for all three variables that formulate synthetic predictor and two variables that formulate synthetic criterion. The two variables "SNS" and "ICAR" load mostly on CV1. CV1 is also strongly related to the variable "choice". On the other hand, both "BNT" and "Payoff" variable loads mostly on CV2. There is a sign difference between SNS, ICAR, and BNT corroborating earlier evidences from the correlational matrix and Figure 2. In addition, BNT has a negative structure coefficient on CV1 and a high positive structure coefficient on CV2. The sign difference indicates opposite relationships with "Choice" variables.

Table 2
Wilcoxon signed-rank test was conducted for participants with high BNT score and Student's t-test was conducted for participants with low BNT scores

BNT score	Condition 1		Condition 2	t	df	p
High	RT in High-Payoff (log)	-	RT in Low-Payoff (log)	256.500		0.025
Low	RT in High-Payoff (log)	-	RT in Low-Payoff (log)	-0.845	31	0.405

Note. Paired Samples T-Test.

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Next, exploratory analysis were performed to observe the relationships between response time, payoff, and numeracy. There is a significant difference in RT between low and high-payoff conditions[5]. Participants' RT was longer (*Mean* = 8.233, *SD* = 0.769) in low-payoff conditions compared to high-payoff conditions

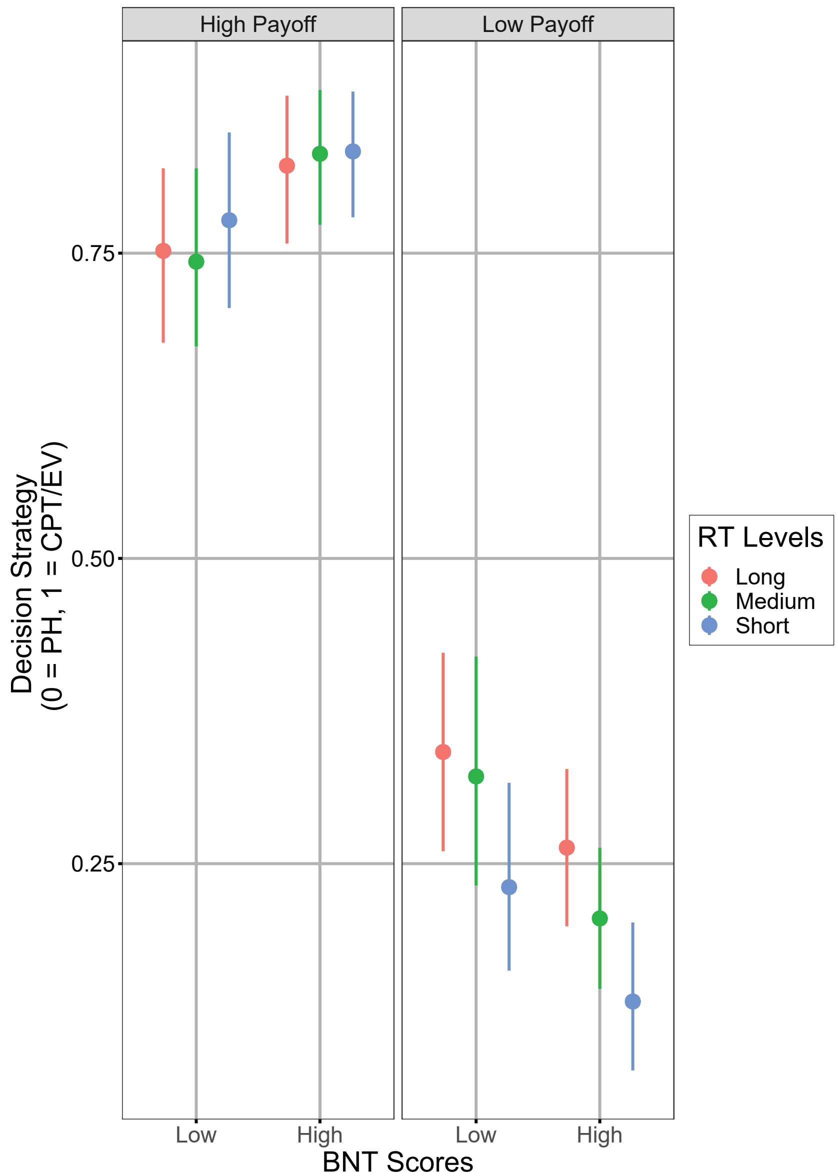

***Figure 3*. Decision strategy as a function of RT and BNT scores under varied payoff condition. Here, BNT – Berlin Numeracy Test; RT – Response time; 0 = PH refers to choices consistent with Priority heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.**

[5] Tables are in Supplementary Materials section.

(*Mean* = 8.131, *SD* = 0.725). Put differently, participants spent comparatively more time over choice problems when the outcome difference between gambles is low regardless of participants' numeracy level. Furthermore, participants with higher BNT scores have significantly longer (*Mean* = 8.241, *SD* = 0.700) RT compared to participants with low BNT scores (*Mean* = 8.096, *SD* = 0.796). Previous results (i.e., Mann-Whitney test results and Figure 2) point out a significant difference in participants' choice strategy based on their numeracy levels in both high and low payoff condition. Analysis of RT data shows a similar trend but only for highly numerate participants. As Table 2 indicates, there is a significant difference in RT between high and low payoff conditions for only participants with high BNT scores, whereas there is no such difference for participants with low BNT scores. Figure 3 effectively corroborates earlier results while validating the interaction effect between payoff, numeracy, and RT across participants.

Notwithstanding, drawing robust conclusion from the aforementioned results is not ideal due to weaker conditional independence. In order to make effective conclusion from the choice data at hand, multi-level regression analysis using Generalised Linear Mixed Model (GLMM) framework was performed (see, McElreath, 2018). In the model, logit function was used as the link function with four fixed effect parameters, two interaction terms, and one random factor. This basic model was declared in the pre-registered form.

Model 1:

$$log\left[\frac{p(choice=1)}{1-p(choice=1)}\right] = \beta_0 + \beta_1(BNT) + \beta_2(SNS) + \beta_3(ICAR) + \beta_4(Payoff) + \beta_5(Payoff:BNT) + \beta_6(Payoff:SNS) + subject_{0s} + e_{si} \quad (2)$$

Where,

$$subject_{0s} \sim N(0, \tau_{00}^2),$$

$$e_{si} \sim N(0, \sigma^2).$$

Here, β_0, β_1,...,β_4 are fixed effect parameters, while β_5, β_6 capture interaction effects. Lastly, error term (e_{si}) and random effect ($subject_{0s}$) is modeled under normal distribution with mean 0 and variance σ^2, and $\tau_{00}{}^2$, respectively. Model 1 has a Nakagawa marginal and conditional R^2 value of 0.41 and .59, respectively with an *AUC* of *ROC* value of 89.69% (Nakagawa & Schielzeth, 2013).

However, Model 1 fails to account for all possible by-subject dependencies. The experiment has multiple observations per combination of participant and payoff conditions, so this variability in the population will also create clustering in the sample, and $subject_{s0}$ alone cannot capture all this variability because it only allows partici

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Table 3
Fixed effects of Model 1

	Estimate	Std. Error	z value	Pr(>| z |)
(Intercept)	0.426	0.238	1.790	0.073
Payoff	2.626	0.208	12.656	<0.001
ICAR	0.374	0.137	2.733	0.006
SNS	0.046	0.025	1.845	0.065
BNT	-0.420	0.330	-1.274	0.203
Payoff:BNT	1.285	0.307	4.191	<0.001
Payoff:SNS	0.036	0.023	1.540	0.124

Table 4
Random effects of Model 1

Groups	Name	Variance	Std.Dev.
Subject	(Intercept)	1.387	1.178

Number of obs: 1752
Groups: Subject = 73

pants to vary around β_0. Hence, random slope was added to allow participants to vary with respect to β_4, our treatment effect. Lastly it is assumed that each participant to have varied preferences among a gamble set; hence Model 1 also lacked a second random effect intercept.

Model 2:

$$log\left[\frac{p(choice=1)}{1-p(choice=1)}\right] = \beta_0 + \beta_1(BNT) + \beta_2(SNS) + \beta_3(ICAR) + \beta_5(RT) +$$
$$\beta_6(Payoff : BNT : RT) + subject_{0s} + Gamble_{0i}$$
$$+ (\beta_4 + subject_{1s})Payoff_i + e_{si} \quad (3)$$

Where,

$$(subject_{0s}, subject_{1s}) \sim N\left(0, \left[\begin{pmatrix} \tau_{00}^2 & \rho\tau_{00}\tau_{11}) \\ \rho\tau_{00}\tau_{11} & \tau_{11}^2 \end{pmatrix}\right]\right),$$

$$gamble_{0i} \sim N(0, \eta_{00}^2),$$
$$e_{si} \sim N(0, \sigma^2).$$

Here, as seen in line 2 of Equation 2, I follow standard assumptions in taking this distribution as a bi-variate normal distribution with a mean of (0, 0) and three free parameters: $\tau_{00}{}^2$ (random intercept variance), $\tau_{11}{}^2$ (random slope variance), and $\rho\tau_{00}\tau_{11}$ (the intercept/slope co-variance). Lastly, the intercept of $gamble_{0i}$ is also drawn from a normal distribution with a mean of 0 and variance of σ^2.

Model 2, compared to Bayes Factor (BF) of 0.005 for Model 1, has a higher BF of 200.87 (Schönbrodt & Wagenmakers, 2018). In addition, Model 2 also has a higher Nakagawa marginal and conditional R^2 value of 0.414 and 0.68 respectively, with a higher *AUC* of *ROC* value of 92.44%. Model 1 has a *RMSE*, and log loss scores of .36 and .404 whereas Model 2 has lower *RMSE*, and log loss scores of .33 and .351, respectively. Moreover, Model 2 has a lower deviance score of 1542.1 compared to the deviance score of 1597.5 of Model 1. In light of the above information, Model 2 is much better at explaining variance with much lower *BIC* and *AIC* scores of 1646.7, and 1570.1 respectively, compared to *BIC*, and *AIC* scores of Model 1 1657.3, and 1613.5, respectively.

Table 5
Fixed effects of Model 2

	Estimate	Std. Error	z value	Pr(>| z |)
(Intercept)	0.746	0.323	2.308	0.021
Payoff	3.026	0.434	6.971	<0.001
Medium RT	-0.177	0.170	-1.043	0.297
Short RT	-0.373	0.178	-2.099	0.036
BNT	-0.374	0.381	-0.980	0.327
SNS	0.042	0.029	1.463	0.143
ICAR	0.403	0.154	2.618	0.009
Payoff:Long RT:BNT	1.625	0.537	3.027	0.003
Payoff:Medium RT:BNT	1.580	0.537	2.940	0.003
Payoff:Short RT:BNT	1.371	0.549	2.497	0.013

Table 6
Random effects of Model 2

Groups	Name	Variance	Std.Dev.	Corr
Subject	(Intercept)	1.789	1.338	
	Payoff	1.682	1.297	0.340
Gamble ID	(Intercept)	0.439	0.662	

Number of obs: 1752
Groups: Subject = 73
Gamble ID = 24.

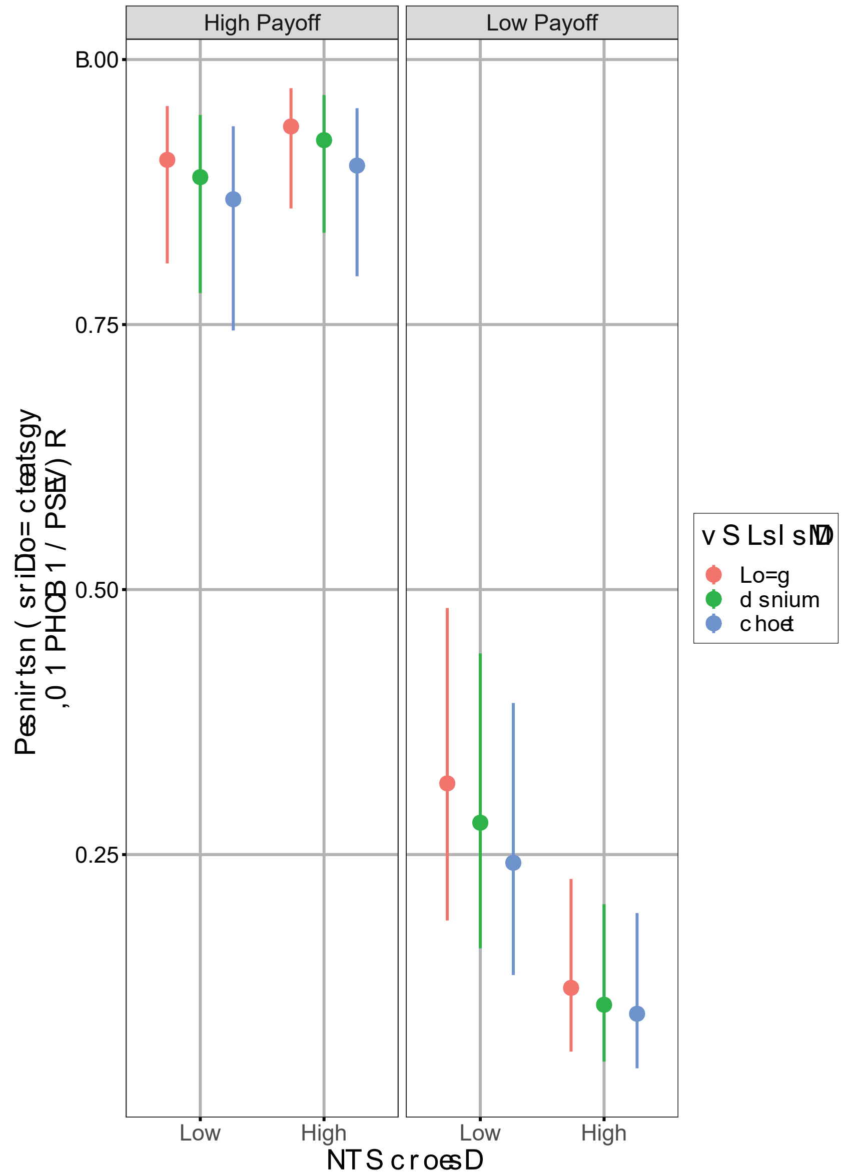

Figure 4. **Predicted decision strategy as a function of RT and BNT scores under varied payoff condition. Here, BNT – Berlin Numeracy Test; RT – Response time; 0 = PH refers to choices consistent with Priority heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.**

As Table 5 indicates, there is significant interaction between participants' objective numeracy and response time in varied payoff conditions. Figure 4 illustrates this interaction more prominently. It was generated by estimating marginal means (predicted values) from Model 2 using *ggeffects* R package (Lüdecke, 2018). High resemblance between simulated data (i.e., Figure 4) and observed data (i.e., Figure 3) indicates robustness of the data collected; at the same time, it attests to the capability of Model 2 to successfully model current data and predict future observations.

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Discussion

The present study examined whether people with higher statistical numeracy, in comparison to people with lower statistical numeracy, strategically employ a more effortful choice strategy to make adaptive choices when the choice problems are meaningful.

Current finding shows that highly numerate individuals seem to follow compensatory decision strategy embodied by CPT/EV significantly more times when the outcome difference between gambles is high compared to less numerate individuals. However, in low-payoff condition, highly numerate individuals change their strategy and opt for a non-compensatory policy that resembles predictions from PH significantly more times than less numerate individuals. This modulation in strategy between two payoff conditions is present for all participants, but the shift in strategy is substantially distinct for highly numerate individuals than individuals with low numeracy, attesting to highly numerate individuals' acuity to changes in payoff structure. The result sufficiently replicates the finding from the original study and the disclosure made in the pre-registration form. This result is also consistent with earlier work (Estrada-Mejia, de Vries, & Zeelenberg, 2016; Ghazal, Cokely, & Garcia-Retamero, 2014; Horn & Freund, 2021; Pachur et al., 2013; Traczyk et al., 2018).

Furthermore, highly numerate individuals did not only made changes in their decision strategy, but also modulated other aspects (i.e., response time) of decision making in accordance with the environment. Current exploratory analysis indicates that highly numerate individuals significantly modulated the amount of time (RT) they spent on each choice problem based on payoff condition; however, individuals with low statistical numeracy did not adjust their response time in relation to payoff condition. Consequently, highly numerate participants strategically employ a more effortful choice strategy to make adaptive choices when the choice problem is meaningful but choose to opt for a heuristics strategy when choices are less meaningful. The current result corroborates with choice data and attests to highly numerate individuals' discernible sensitivity to payoff structure changes.

Apart from measuring objective numeracy, two other scales were also used to measure subjective numeracy and fluid intelligence. Results show that objective and subjective numeracy explains different aspects of human decision-making (Peters & Bjalkebring, 2015). Individuals with high BNT scores, on average, opted for a strategy predicted by PH in low-payoff conditions, opposite of individuals with high SNS scores. On the other hand, on average, individuals with low SNS scores opted for a strategy predicted by PH in low-payoff conditions, opposite of individuals with high BNT scores. Existing literature suggests that individuals with high objective numer-

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acy are better equipped to do number comparisons, operations, and calculations, whereas subjective numeracy has been linked to emotional reactions to numbers. Individuals with higher subjective numeracy, unsurprisingly, have more confidence in their ability to perform effectively in numeric tasks and follow EV maximization policy irrespective of the payoff structure (Peters & Bjalkebring, 2015; Traczyk et al., 2018). On the contrary, numerate participants are more sensitive to changes in the environment and make normatively superior decisions adaptively. This contrast helps to explain quantitative differences in predictions from subjective and objective numeracy measures. Although contrary to earlier studies, there is a negligible correlation between SNS and BNT in our study (Sobkow, Olszewska, & Traczyk, 2020; Traczyk et al., 2018). Authors of SNS argued that SNS could replace BNT or could be used as a proxy of BNT (Fagerlin et al., 2007). However, results from the original study and the current study indicate that both scales predict different outcomes hence can not be replaced or be used interchangeably.

Nevertheless, there are some limitations I need to acknowledge. From the data, I could not conclude whether less numerate individuals were making choices that are more consistent with predictions made by CPT/EV theory or they made random choices, given choices are less meaningful in low-payoff conditions. Put differently, for choices in which outcome differences between gambles are low, less numerate participants could have been more inconsistent and switched between strategies (i.e., CPT/EV, PH, random), but such questions are beyond the current experimental purview. Future research can look into this matter. The current replication study used a within-participant design. In future studies, I intend to conduct further experiments with between-participant design to establish the causal relationship between adaptive behavior and numeracy. The current study was conducted in the gain domain. Hence current gambles used in the study may not capture risk attitude of participants adequately. Future work should use gambles from the mixed domain. Lastly, participants had the luxury to spend as much time as they wished for each problem, but in reality, there are always costs associated with time. Hence, I intend to further explore whether numerate individuals continue to follow EV maximization strategy in meaningful circumstances under time pressure.

Conclusion

The current study sufficiently demonstrated that subjective and objective numeracy made quantitatively different predictions under risk. Importantly, I successfully replicated the effect where objectively numerate decision-makers are more sensitive to changes in payoff structure and modulate their strategy to an effortful choice strategy in order to make adaptive choices when the choice problem is meaningful. In summary, I demonstrated that people with higher statistical numeracy, compared to people with lower statistical numeracy, strategically employ more energy-intensive choice strategies to make adaptive choices when the choice problem is meaningful; otherwise, numerate individuals use less effortful heuristic strategies.

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10.1.1 Supplementary Materials

1 Supplementary Materials

Table 1: Descriptive table of individual measures

	BNT	SNS	ICAR	CPT/EV choices in high-payoff problems	CPT/EV choices in low-payoff problems	RT in high-payoff problems	RT in low-payoff problems
Mean	1.7	34.4	2.1	0.8	0.2	3777	4278.2
Std.Dev	1.3	6.4	1.3	0.2	0.2	1866.7	2838.2
Min	0	14	0	0.2	0	996	295
Q1	0	30	1	0.6	0.1	2497	2791
Median	2	35	2	0.8	0.2	3383	3878
Q3	3	39	3	1	0.4	4593	5513

Table 2: Mann-Whitney test was conducted to illustrate the difference in choice strategy in high-payoff choice problems by participates with low and high BNT scores.

	W	p	Rank-Biserial Correlation
Choice	118852.500	0.006	0.072

Note. For the Mann-Whitney test, effect size is given by the rank biserial correlation.

Table 3: Mann-Whitney test was conducted to illustrate the difference in choice strategy in low-payoff choice problems by participates with low and high BNT scores.

	W	p	Rank-Biserial Correlation
Choice	72242.500	0.003	-0.090

Note. For the Mann-Whitney test, effect size is given by the rank biserial correlation.

1.1 Interaction between response time (RT) and Payoff

Table 4: Descriptive table of participants' RT in varied payoff condition.

	Condition	N	Mean	SD	SE
RT (log)	High Payoff	949	8.131	0.725	0.024
	Low Payoff	803	8.233	0.769	0.027

Table 5: Test of Normality (Shapiro-Wilk) for participants' RT in varied payoff condition.

		W	p
RT (log)	High Payoff	0.980	$< .001$
	Low Payoff	0.976	$< .001$

Note. Significant results suggest a deviation from normality.

Table 6: Mann-Whitney U test of RT in varied payoff condition.

	W	df	p
RT (log)	344856.000		$< .001$

1.2 Interaction between response time (RT) and BNT levels

Table 7: Descriptive table of RT between participants with high and low BNT scores

	Condition	N	Mean	SD	SE
RT (log)	High BNT	984	8.241	0.700	0.022
	Low BNT	768	8.096	0.796	0.029

Table 8: Test of Normality (Shapiro-Wilk) for RT between participants with high and low BNT scores

		W	p
RT (log)	High BNT	0.983	$< .001$
	Low BNT	0.971	$< .001$

Note. Significant results suggest a deviation from normality.

Table 9: Mann-Whitney U test for RT between participants with high and low BNT scores.

	W	df	p
RT (log)	407826.000		0.004

2 Summary of Model 1

Table 10: Model performance

AIC	BIC	BF	R2_conditional	R2_marginal	RMSE	Log_loss	ROC
1613.519	1657.268	0.005	0.586	0.411	0.359	0.404	89.69%

3 Summary of Model 2

Table 11: Model performance

AIC	BIC	BF	R2_conditional	R2_marginal	RMSE	Log_loss	ROC
1570.103	1646.662	200.86	0.678	0.414	0.334	0.351	92.44%

Table 12: Contrast table of interaction effect calculated from marginal means of the simulated data from Model 2.

RT Levels (log)	Contrast	Estimate	SE	z.ratio	p.value
Long	Low BNT, Low-Payoff	-1.142	0.284	-4.021	< 0.001
	High BNT, Low-Payoff	-2.329	0.321	-7.26	< 0.001
	Low BNT, High-Payoff	1.883	0.324	5.806	< 0.001
	High BNT, High-Payoff	2.322	0.367	6.324	< 0.001
Medium	Low BNT, Low-Payoff	-1.32	0.289	-4.561	< 0.001
	High BNT, Low-Payoff	-2.483	0.327	-7.596	< 0.001
	Low BNT, High-Payoff	1.706	0.322	5.297	< 0.001
	High BNT, High-Payoff	2.122	0.358	5.929	< 0.001
Short	Low BNT, Low-Payoff	-1.516	0.292	-5.189	< 0.001
	High BNT, Low-Payoff	-2.575	0.344	-7.489	< 0.001
	Low BNT, High-Payoff	1.51	0.322	4.689	< 0.001
	High BNT, High-Payoff	1.821	0.349	5.224	< 0.001

Table 13: Gambles used in the study

Gamble 1	Gamble 2	CPT/EV	PH
Win $3.0 with probability of 17%	Win $56.7 with probability of 5%	B	A
Win $3.0 with probability of 29%	Win $56.7 with probability of 9%	B	A
Win $56.7 with probability of 5%	Win $3.0 with probability of 17%	A	B
Win $56.7 with probability of 9%	Win $3.0 with probability of 29%	A	B
Win $5.4 with probability of 52%	Win $56.7 with probability of 29%	B	A
Win $3.0 with probability of 94%	Win $56.7 with probability of 29%	B	A
Win $31.5 with probability of 29%	Win $3.0 with probability of 52%	A	B
Win $56.7 with probability of 29%	Win $5.4 with probability of 52%	A	B
Win $3.0 with probability of 94%	Win $31.5 with probability of 52%	B	A
Win $5.4 with probability of 94%	Win $56.7 with probability of 52%	B	A
Win $31.5 with probability of 52%	Win $3.0 with probability of 94%	A	B
Win $56.7 with probability of 52%	Win $5.4 with probability of 94%	A	B
Win $17.5 with probability of 52%	Win $56.7 with probability of 17%	B	A
Win $9.7 with probability of 52%	Win $31.5 with probability of 17%	B	A
Win $5.4 with probability of 29%	Win $9.7 with probability of 17%	B	A
Win $31.5 with probability of 29%	Win $56.7 with probability of 17%	B	A
Win $3.0 with probability of 29%	Win $5.4 with probability of 17%	B	A
Win $3.0 with probability of 52%	Win $9.7 with probability of 17%	B	A
Win $17.5 with probability of 17%	Win $3.0 with probability of 94%	A	B
Win $9.7 with probability of 17%	Win $5.4 with probability of 29%	A	B
Win $56.7 with probability of 17%	Win $17.5 with probability of 2%	A	B
Win $9.7 with probability of 17%	Win $3.0 with probability of 52%	A	B
Win $5.4 with probability of 17%	Win $3.0 with probability of 29%	A	B
Win $31.5 with probability of 17%	Win $5.4 with probability of 94%	A	B

10.2 Second Published Manuscript

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Conditionality of adaptiveness: Investigating the relationship between numeracy and adaptive behavior☆

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ABSTRACT

Recent studies have illustrated that individuals with higher numeracy are more likely to make adaptive choices than individuals with lower numeracy. Highly numerate individuals can consistently make normatively superior choices by maximizing expected value (EV) in meaningful choice problems (high-payoff condition). However, in trivial problems (low-payoff condition), they can also adaptively change their strategy to make good enough choices and not follow a normatively superior strategy. Upon inspection of choice problems used in earlier studies, it was revealed that payoff was not the only varying factor between the two payoff conditions. Therefore, it is unclear whether payoff conditions alone can provide sufficient context for adaptive modulation in decision strategy. In two pre-registered studies (N = 343), we tested numerate individuals' adaptiveness under high- and low-payoff conditions addressing the limitations of earlier studies. Results revealed that the presence of two payoff conditions together did not initiate adaptive strategy selection, regardless of participants' numeracy. Instead, numerate individuals, compared to less numerate individuals, consistently made more EV-consistent choices in both payoff conditions. We identified that the change in EV consistency across payoff conditions was influenced more by the absolute difference than the relative difference in the expected reward.

1. Introduction

Making superior choices under risk demands flexibility in reactions to dynamically changing task demands (Gigerenzer, Todd, & ABC Research Group, 1999; Payne, Bettman, & Johnson, 1993). Multiple theories (i.e., Drift diffusion model, Decision field theory, Dynamic signal detection theory, to name a few) and empirical results in decision-making advocate that extensive deliberation always leads to better and superior choices (Bogacz, Wagenmakers, Forstmann, & Nieuwenhuis, 2010; Busemeyer & Townsend, 1993; Pleskac & Busemeyer, 2010; Ratcliff & McKoon, 2008; Ratcliff, Smith, Brown, & McKoon, 2016). However, when relatively less is at stake, one has the luxury of not taking the best action available and instead choosing an action that is just good enough. Every day we dwell on decisions that require different levels of deliberation as they differ in the extent of their consequences. For example, if one's personal goal is to make a good impression during an interview for a desired job, the decision to take an umbrella on a cloudy day is straightforward, and deliberation on its cost and benefit is unnecessary. However, if the personal goal is to have a good time with an old friend, we may seriously consider whether it is worth carrying the cumbersome umbrella while accepting the risk of leaving it in a restaurant or getting wet on the way. This example illustrates that a numerically proficient

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person may carefully choose an option that maximizes their expected value (EV) when the difference in value between two options is sufficiently large and stakes are meaningful (i.e., getting the job vs. not getting the job). On the other hand, the same person may not carefully consider all possible outcomes when the stakes are more trivial (i.e., having a good time vs. the risk of getting wet). Instead, one may choose any option that seems just good enough, thereby saving time and effort. In a similar vein, Traczyk, Sobkow, et al. (2018) demonstrated that more numerate individuals could better understand the structure of the environment than less numerate individuals. They observed that more numerate individuals can employ adaptive strategies to maximize value when it is meaningful (i.e., high-payoff condition) and save time and effort when the problems are trivial (i.e., low-payoff condition). In the current study, we examined such adaptive modulation in strategy in a more controlled task environment. More specifically, we aimed to evaluate whether more numerically proficient individuals are more likely to follow the EV maximization model when the relative difference in value is significant (i.e., high-payoff condition) and, on the other hand, adaptively change their plan to an EV inconsistent strategy when the relative difference in value is small (i.e., low-payoff condition). Inconsistent with earlier findings, we discovered that individuals do not adaptively modulate their strategy following changes in payoff conditions regardless of their numeric proficiency. Instead, individuals focus on the absolute difference in value when making decisions.

2. Theoretical background

2.1. Numeracy and superior decision making

Understanding numerical information (i.e., statistical numeracy) and its effective use in everyday situations is the key to making good decisions (Ancker & Kaufman, 2007; Cokely et al., 2018; Paulos, 1988; Reyna & Brainerd, 2007). Since the publication of the first psychological tests measuring numeracy, the area of research on this construct has developed rapidly (Garcia-Retamero, Sobkow, Petrova, Garrido, & Traczyk, 2019; Nelson, Reyna, Fagerlin, Lipkus, & Peters, 2008; Reyna, Nelson, Han, & Dieckmann, 2009). Numeracy can be defined as a measure that evaluates individual proficiency in probabilistic and statistical computations (Schwartz, Woloshin, Black, & Welch, 1997). Researchers are focused on understanding psychological mechanisms that differentiate individuals with low and high numeracy. These efforts would be helpful in developing methods for communicating numerical information to people with low numeracy, which would allow such people to make more informed decisions. For example, the results of studies conducted so far indicate that people with high numeracy, compared to people with low numeracy, make normatively better decisions (i.e., those that are consistent with the predictions of the expected value maximization model) and are less sensitive to various cognitive biases (Liberali, Reyna, Furlan, Stein, & Pardo, 2012; Peters & Levin, 2008; Reyna & Brainerd, 2007). As a consequence, in everyday functioning, higher numeracy may manifest itself in better overall health (Garcia-Retamero, Andrade, Sharit, & Ruiz, 2015) and in greater wealth (Estrada-Mejia, de Vries, & Zeelenberg, 2016; Tang, 2021).

Among the potential psychological mechanisms that are responsible for differences in decision making between people with high and low numeracy, we can distinguish: (1) more linear weighing of outcomes and probabilities (Millroth & Juslin, 2015; Patalano, Saltiel, Machlin, & Barth, 2015; Petrova, van der Pligt, & Garcia-Retamero, 2014; Schley & Peters, 2014; Traczyk & Fulawka, 2016), (2) longer deliberation on the problem (Ghazal, Cokely, & Garcia-Retamero, 2014; Petrova, Garcia-Retamero, Catena, & van der Pligt, 2016), (3) elaborative and more thorough information processing based on heuristics (Cokely & Kelley, 2009), (4) creating a gist representation of the decision problem (Broniatowski & Reyna, 2018; Reyna & Brainerd, 2008), which leads to a more accurate assessment of the accuracy of the decision made (Barrafrem, Västfjäll, & Tinghög, 2021; Garcia-Retamero, Cokely, & Hoffrage, 2015), and (5) consideration of important affective information (e.g., fear of negative consequences of choices) in the decision-making process (Grohmann, Kouwenberg, & Menkhoff, 2015; Peters, 2012; Peters et al., 2006; Traczyk & Fulawka, 2016; Traczyk, Lenda, et al., 2018).

In this sense, people with high numeracy do not only make good decisions due to their greater efficiency in counting, more extensive mathematical knowledge, or acquired skills in this field. Instead, highly numerate people are more sensitive to changes in the probability scale and spend more time processing a decision problem, which in turn allows them to create a general, affect-rich, and accurate mental representation of the decision problem. Such cognitive faculties can facilitate adaptive decision-making among people with high numeracy.

2.2. The adaptive role of numeracy

When making risky decisions, our choices can be shaped by different factors (Baron, 2008). The simplest model that is often used to evaluate such factors is the EV model. It assumes that the decision-maker maximizes EV, understood as the sum of the products of payoffs and probabilities assigned to them (for exact formula, see Appendix 1). Existing findings indicate that there is a positive relationship between numeracy and more frequent use of choice strategies based on maximizing EV (Pachur & Galesic, 2013). Interestingly, Cokely and Kelley (2009) showed that despite the positive relationship between numeracy and choices maximizing EV, the decision process of people with high numeracy was preceded by operations unrelated to the calculation of EV. They concluded, employing the retrospective verbalizations method, that participants used an elaborative heuristic search to make their judgment. These verbalizations have manifested, for example, in more frequent transformations of probabilities, focusing on the most and least favorable outcomes, or considering the risk associated with other options. This result points towards two conclusions. On one hand, people with high numeracy can better perform a more complex mathematical operations resulting in choices maximizing EV—they are more sensitive to changes in EV (Jasper, Bhattacharya, Levin, Jones, & Bossard, 2013). On the other hand, their choices can be based on accurate heuristics. In this sense, more numerate people can have a greater repertoire of decision strategies that will be adaptively selected depending on the structure and requirements of the decision problem (Payne, Bettman, & Johnson, 1988).

Traczyk, Sobkow, et al. (2018) tested the above conclusion using a series of 24 two-outcome choice problems, where choice problems were divided into high-payoff (EV ratio is around 5) and low-payoff condition (EV ratio is around 1). Decision problems were considered less important in the low-payoff condition because, regardless of participants' choice, they would earn relatively similar payoffs. However, choice problems in the high-payoff condition were considered to be more important because selecting a gamble with a higher EV, on average, will lead to a significantly higher payoff. The choice problems were designed to distinguish between compensatory (i.e., Cumulative prospect theory/EV maximization) and non-compensatory (i.e., Priority Heuristics) strategies, which allowed the authors to track modulation (if any) in strategies across the two payoff conditions. The first model—cumulative prospect theory (CPT; Tversky & Kahneman, 1992)—posits that the decision-maker weighs the subjective representation of outcomes with a subjective representation of probabilities (in the form of decision weights). This operation is performed for each gamble (prospect), where the decision maker selects the prospect with the highest subjective value. Also, predictions of CPT, simulated with the standard parameters from the seminal paper of Tversky and Kahneman (1992), were consistent with the EV maximization model. Hence, when participants select any gamble consistent with the predictions of CPT, participants inevitably choose the same gamble with a higher EV (i.e., EV maximization strategy). The second model—priority heuristics (PH; Brandstätter, Gigerenzer, & Hertwig, 2006) assumes that decision-makers do not perform transformations aimed at calculating or approximating EV, rather they sequentially compare the properties of the decision problem by focusing on the least attractive payoff, its probability, and the most attractive payoff. When the difference between these values is significant, the decision-maker applies the stopping rule and makes a choice.

Previous results in Traczyk, Sobkow, et al. (2018) suggest that, in meaningful problems (i.e., high-payoff condition), individuals with higher objective numeracy maximized EV and made choices consistent with the predictions of CPT. However, in trivial problems (i.e., low-payoff condition), where the EV ratio was low and the potential payoffs in the two gambles were comparable regardless of the decision made, they made choices inconsistent with EV maximization model and in line with the predictions of PH, understood here as the adaptive modulation in choice strategy. Put simply, highly numerate individuals are more sensitive to changes in EVs, allowing them to maximize expected payoff when the decision problems are meaningful. On the other hand, highly numerate individuals do not use an EV maximization strategy when expected payoffs are comparable in trivial problems. This result was successfully replicated (Mondal, 2021).

However, these previous studies suffer from some limitations. Choice problems used in earlier studies were not well controlled. Contrary to the conclusion, payoff is not the only varying factor between high- and low-payoff conditions. The absolute difference in expected values between two options has a skewed distribution across the two payoff conditions —that is, choice problems in the low-payoff condition have consistently lower absolute differences in expected values compared to the high-payoff condition. Furthermore, the difficulty level[1] between two payoff conditions is also not consistent (Pachur et al., 2013). As a result, adaptive modulation in decision strategy between two-payoff conditions can not only be attributed to varied EV ratios embodied by payoff conditions. Put differently, it is not clear whether payoff conditions alone can provide sufficient context necessary for highly numerate individuals to make better adaptive modulation in decision strategy compared to less numerate individuals.

3. Experiment 1

In the first experiment, we aim to test adaptive strategy selection between highly numerate and less numerate individuals with more evenly-distributed choice problems. We expect, on average, highly numerate individuals, compared to less numerate individuals, to make more EV consistent choices in the high-payoff condition (i.e., important problems) but not in the low-payoff condition (i.e., trivial problems) by recognizing the relative difference in payoff distribution across the two payoff conditions. Here, EV consistency refers to choices that, if made, maximize expected value. In the same vein, EV inconsistency refers to choices that, if made, do not maximize expected value. That includes choices that, on average, are random (i.e., EV consistency around 50%). Therefore, participants' choices would be considered adaptive if participants, on average, followed the EV consistent strategy in the high-payoff condition and changed their strategy to EV inconsistent choices in the low-payoff condition.

The decision to use the expected value maximization model to measure changes in decision strategy hinges on the normative quiddity of the EV maximization model.[2] Even though numerous descriptive models of decision making (e.g., cumulative prospect theory, TAX, priority heuristic, etc.) have been shown to explain and predict choices under risk better than the EV model, we

[1] Difficulty can be defined in terms of the similarity between options (e.g., similar EVs; Pachur, Hertwig, Gigerenzer, & Brandstätter, 2013). In other words, the similarity of EVs between options makes a choice problem more difficult, increasing the error rate and decision time. However, Alós-Ferrer and Garagnani (2022) demonstrated that these chronometric and psychometric effects are a function of the differences in expected utility and not expected value. For the choice problems used in the current study, the prediction made by the expected utility theory with a power utility function and expected value model are very similar (i.e., 97.2%; for more details, see Section 8 from the Appendix). Hence, we chose to define difficulty in terms of similarity in EVs between options.

[2] We chose the EV maximization model to monitor changes in decision strategy, assuming that a decision maker is indifferent between two lotteries with the same EV. In other words, we assumed that lottery A (10% chance of getting 300 $; otherwise 0) and lottery B (1% chance of getting 3000 $; otherwise 0) are the same. However, a decision maker who is minimally risk averse will consistently prefer lottery A over lottery B due to the concavity of the utility function. It has been demonstrated that risk neutrality is the optimal strategy (i.e., following EUT) in tasks with multiple choices with modest stakes because risk aversion in small stake choices results in absurdly high levels of risk aversion over large stakes (Rabin, 2000). Although evidence of risk-averse behavior can be observed over modest stake choices, Rabin and Weizsäcker (2009) argue that risk aversion observed in small stakes laboratory tasks could be attributed to narrow bracketing.

decided to incorporate the latter model as a benchmark for optimal decision making. According to the proposition of normativity, choosing the option with a higher EV will provide the decision maker with a better average outcome in the long run, over an infinite number of independent yet structurally identical repetitions (Baron, 2008). Nevertheless, we have also simulated prominent expectation models among other positive decision making models to observe the similarity in predictions across 72 choice problems. We did not find any statistical difference in the prediction between the EV maximization model, CPT, and Expected Utility Theory (EU; see, Figure 1 from the Appendix). Besides, the EV maximization model gives us a tool to measure changes in optimality in accordance with the changes in relative difference (i.e., two payoff conditions). In other words, the EV maximization model will allow us to monitor whether participants maximize EV regardless of the payoff conditions or whether they adaptively modulate between optimal and sub-optimal strategies in accordance with the payoff condition. This definition of adaptiveness was adopted from earlier studies (Mondal, 2021; Traczyk, Sobkow, et al., 2018). Lastly, given that participants adaptively modulate their strategy between two payoff conditions, we also anticipate that the relative magnitude of change will be larger for numerate individuals than for less numerate individuals.

3.1. Materials and procedure

We presented each participants with 36 high-payoff and 36 low-payoff choice problems in a random order and recorded their preference and response time (RT). In addition, we also measured participants' numeracy using the Berlin Numeracy Test (BNT; Cokely, Galesic, Schulz, Ghazal, & Garcia-Retamero, 2012). The presentation of choice problems and numeracy scale was counterbalanced. Pre-registration, experimental procedure, choice problems, data used for analysis, and complete analysis (R markdown file) of Experiment 1 has been posted on the Open Science Framework (https://osf.io/p8av4/).

3.1.1. Objective statistical numeracy

We used the BNT to measure objective statistical numeracy and risk literacy. A computerized version of the traditional paper and pencil format of the BNT was presented to participants. Possible scores ranged between 0 to 4 points, with higher scores indicating higher objective statistical numeracy. We planned to divide participants into high numeracy (BNT $\geq$ 2) and low numeracy groups (BNT $\leq$ 1) based on their performance on the BNT scale.

3.1.2. Choice problems

We developed 72 new binary choice problems consisting of two-outcome gambles distributed evenly between the high- and low-payoff condition in the gain domain. The distribution of EV ratios (see, Appendix 1.1 for exact formula of EV ratio) between the two payoff conditions was adopted from Traczyk, Sobkow, et al. (2018). In the low-payoff condition, as the name suggests, the EV difference between options is relatively low (i.e., the EV ratio between options is 1–2), indicating a relatively insignificant difference in value between the two options. Conversely, the EV difference between options is relatively large (i.e., EV ratio between options is 5–6) in the high-payoff condition, indicating a substantial difference in value between the two options (Mondal, 2021; Pachur et al., 2013; Traczyk, Sobkow, et al., 2018).

In earlier studies, the EV ratio is not the only factor that varied between two payoff conditions (Mondal, 2021; Traczyk, Sobkow, et al., 2018). Choice problems used by Traczyk, Sobkow, et al. (2018) were not controlled adequately across the two payoff conditions. For example, as Fig. 1A suggests, the Absolute-EV-Difference (AED; see, Appendix 1.2 for exact formula) between options was not controlled. AED between options is always higher for choice problems in the high-payoff condition, whereas AED between options is always lower for choice problems in the low-payoff condition. This skewness in AED distribution resulted in an asymmetric trade-off between EV consistent and EV inconsistent choices. On average, there is no difference in total reward earned by participants when they fail to maximize EV compared to when they maximize it in the low-payoff condition. On the contrary, as Fig. 1B suggests, there is, on average, a significant difference in total reward earned by participants when they fail to maximize EV compared to when they do maximize it in the high-payoff condition. These differences in reward distribution across the two payoff conditions are referred to as an asymmetric trade-off. The asymmetric trade-off is a property of the choice problems and is not used in its conventional sense (i.e., not an attribute of participants).

In the current study, we followed the same principle to develop the choice problems and tried to control as many factors as possible (i.e., variance, AED between options, and asymmetric trade-off) to let only the EV ratio vary between the two conditions (see Table 1). We controlled outcomes and AED by restricting them to 100 and 50, respectively, in both payoff conditions. As a consequence, regardless of the payoff condition, participants will earn significantly less if they decide to make EV inconsistent choices compared to EV consistent choices. There is now, unlike in earlier studies, a trade-off present if one decides to change the decision strategy. Furthermore, the relative position of the choice problem is randomized for each participant. Also, we counterbalanced option placement as well (i.e., option A is not option A for each participant). All control measures were taken to vary only EV ratios between two payoff conditions across all participants. That is, the difference between options, across two payoff conditions, did not differ in variance (p = .925), outcome (p = 1.00), and probability (p = .722). In addition, variance (p = .805), outcome (p = .940), probability (p = .232), and expected value (p = .218) of option A did not differ significantly from option B over 72 choice problems to ensure that the change in participants' response is not based on any other factors other than EV ratio of 72 choice problems (see, Tables 1 and 4 from the Appendix for more details on control measures).

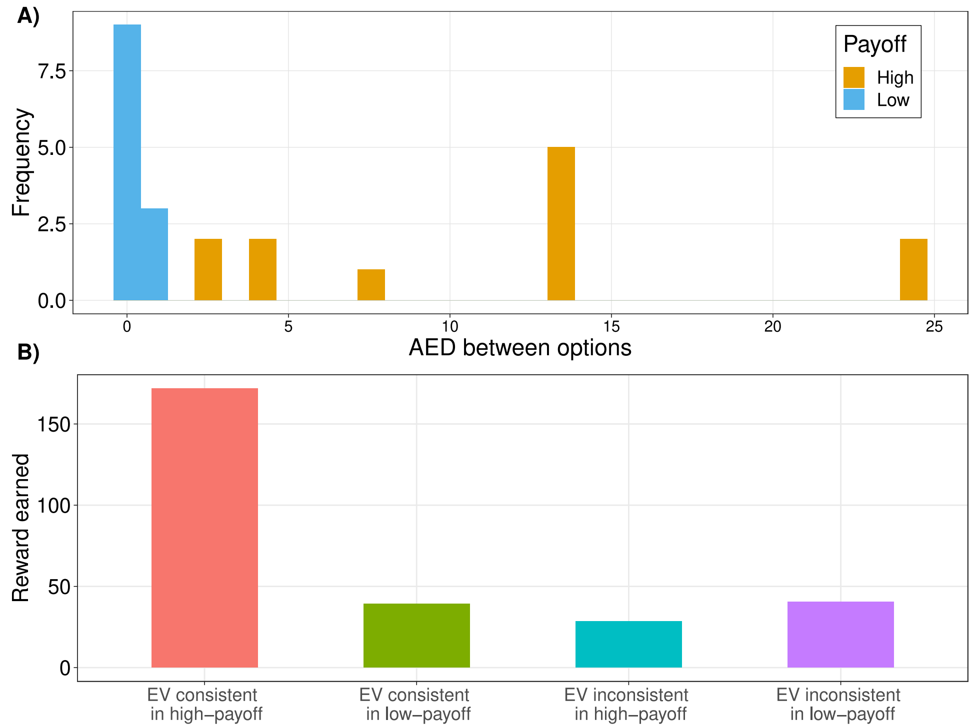

Fig. 1. Distribution of Absolute EV Difference (AED) and possible earning is plotted across two payoff conditions for choice problems taken from Traczyk, Sobkow, et al. (2018). (A) The distribution of Absolute EV Difference (AED) between options is plotted across the two payoff conditions. (B) Possible earning of reward is plotted as a function of different choice strategies. Here, EV consistent refers to choices consistent with the EV maximization model, and EV inconsistent indicates choices inconsistent with the EV maximization model.

Table 1
Composition of the newly developed 72 choice problems. High-payoff and low-payoff condition constitute EV ratio of 5–6 and 1–2, respectively. Large AED refers to the AED difference of 33.3–50; Medium AED refers to the AED difference of 16.6–33.2; Small AED refers to the AED difference of 0–16.6.

AED	Payoff	Total number	Representative example	AED between options	EV ratio
Large	High	12	(A) 9% chance to receive 93 points (B) 63% chance to receive 70 points	35.73	5.27
Large	Low	12	(A) 55% chance to receive 92 points (B) 99% chance to receive 90 points	38.5	1.76
Medium	High	12	(A) 8% chance to receive 59 points (B) 54% chance to receive 45 points	19.58	5.15
Medium	Low	12	(A) 33% chance to receive 61 points (B) 79% chance to receive 50 points	19.37	1.96
Small	High	12	(A) 4% chance to receive 58 points (B) 99% chance to receive 13 points	10.55	5.55
Small	Low	12	(A) 81% chance to receive 39 points (B) 31% chance to receive 71 points	9.58	1.44

3.2. Participants

Ninety-five volunteers participated (*mean age* = 26.32 years, *SD* = 6.81) in an online study for a half-hourly compensation of 2.56 GBP (equivalent to approximately 3.5 USD). The sample size was calculated using G*Power software to obtain a statistical power of .95 with a significance level (α) of 0.05 in order to detect an effect size (d) of 0.684 (medium effect size was noticed in the previous study; Mondal, 2021). Participants were recruited via the Prolific platform. They were instructed that the aim of the study is to measure the cognitive abilities of people belonging to different social and demographic groups (see, Appendix 10 for more detailed instructions). Furthermore, participants were told that they would be asked to solve a few tasks and questionnaires, and they would be compensated based on their performance in the task. As a bonus, participants were informed that for every 1000 points, they would receive an additional 1 GBP on top of the flat fee. Lastly, participants were made aware that they could stop the task at any time. However, if they do so, they would not be able to continue the study further and would not receive any compensation.

Table 2
Descriptive table of individual measures.

	BNT	EV consistency		RT (log)	
		High payoff	Low payoff	High payoff	Low payoff
Median	1.000	0.861	0.806	8.040	8.110
Mean	1.126	0.846	0.797	8.054	8.104
SD	1.240	0.135	0.130	0.523	0.523
MAD	1.000	0.083	0.083	0.280	0.290
Variance	1.537	0.018	0.017	0.274	0.274

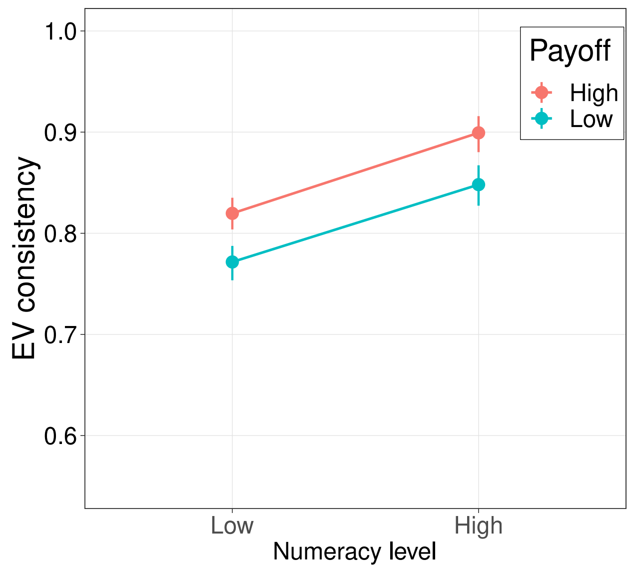

Fig. 2. EV consistency plotted as a function of numeracy score across two payoff conditions.

3.3. Results

3.3.1. Numeracy

The mean score of participants in the BNT scale is 1.126 with *SD* of 1.24. Out of 95 participants, 32 participants are part of the high numeracy group (BNT $\geq$ 2), and 63 participants are part of the low numeracy group (BNT $\leq$ 1) (see Table 2).

3.3.2. Choice problems

Mean EV consistency between the two payoff conditions were analyzed using the paired samples t-test.[3] Mean EV consistency is significantly higher, $t(94) = 5.72$, $p < .001$, $d = .59$, in the high-payoff condition ($M = 0.85$, $SD = 0.14$), consistent with earlier results, than mean EV consistency in the low-payoff condition ($M = 0.80$, $SD = 0.13$). In contrast, mean RT in the low-payoff condition ($M = 8.10$, $SD = 0.52$) is significantly higher, $t(94) = 3.71$, $p < .001$, $d = .38$, than mean RT in the high-payoff condition ($M = 8.05$, $SD = 0.523$).

Next, we used the independent sample Mann–Whitney U-test to compare EV consistency in the high-payoff condition between high and low numeracy groups. The results suggest that participants with high numeracy followed EV consistent choices, consistent with our hypothesis and earlier results, significantly more times, $U = 1459$, $p < .001$, $r_b = .45$, in the high-payoff condition ($M = 0.90$, $SD = 0.13$) compared to less numerate participants ($M = 0.82$, $SD = 0.13$). However, as Fig. 2 suggests, this trend continues, inconsistent with our hypothesis and earlier results, in the low-payoff condition as well. Highly numerate individuals followed EV consistent choices ($M = 0.85$, $SD = 0.12$), significantly more often, $U = 1412.5$, $p < .001$, $r_b = .40$, in the low-payoff condition compared to less numerate individuals ($M = 0.77$, $SD = 0.13$). Furthermore, we used the independent sample t-test to measure the magnitude of change in decision strategy (μ_{md}; see Appendix 2) between the high numeracy and low-numeracy group. Results suggest that there is no significant difference in μ_{md}, $t(93) = 0.17$, $p = .86$, $d = .04$, between highly numerate individuals ($M = 0.051$, $SD = 0.073$) and less numerate individuals ($M = 0.048$, $SD = 0.089$).

Following this result, we used Hayes' PROCESS-macro analysis (4.0.1) to test whether the nature of the relationship between payoff and EV consistency varies as a function of AED (Hayes, 2012). In this regression-based analysis, we tested whether the

[3] We reanalyzed the current choice data similar to how it was analyzed in the study done by Traczyk, Sobkow, et al. (2018). The conclusions of the current study are robust to different ways of analyzing the data. For more details, see Table 7 & 10 from the Appendix.

interaction of two predictors (e.g., X and W) significantly improved the model fit indexed by the change of R^2. The significant interaction indicates that the effect of predictor X on dependent variable Y is moderated by predictor W. We mean-centered the continuous variable (i.e., AED) and dummy coded the categorical variables (i.e., Payoff and EV consistency) to avoid high multi-collinearity with the interaction term. Results from the moderation analysis suggest that there is a significant interaction, $\beta = -0.014$, $Z = -2.97$, $p = .003$, between payoff and AED on EV consistency. However, there is no significant direct effect, $\beta = 0.129$, $Z = 1.86$, $p = .063$, of payoff conditions on EV consistency. On the other hand, AED has a significant direct effect, $\beta = 0.035$, $Z = 14.60$, $p < .001$, on EV consistency. In addition, We calculated the Johnson-Neyman interval to further prove the interaction effect of AED between payoff and EV consistency. As Fig. 3 A suggests, when AED difference between options crosses the [23.11] point, the slope of the Payoff difference becomes insignificant ($p > .05$). Put differently, when AED between options crosses the [23.11] point, there is no significant difference in EV consistency between the two payoff conditions.

3.4. Summary

The results at hand indicate that there is a positive relationship between numeracy and EV consistency across the two payoff conditions (Mondal, 2021; Traczyk, Sobkow, et al., 2018). However, the presence of the high-payoff and low-payoff condition together does not necessarily initiate adaptive strategy selection, regardless of participants' numeracy, when choice problems are evenly distributed. Highly numerate individuals changed their decision strategy between the high- and low-payoff condition, but the change does not embody adaptive strategy selection. Highly numerate individuals, compared to less numerate individuals, made EV consistent choices in the high-payoff condition. However, they made more EV consistent choices, contrary to our expectations and earlier results, in the low-payoff condition as well.

Compared to earlier results, the magnitude of difference in EV consistent choices between the two payoff conditions is also relatively small. In addition, there is no significant difference in the degrees of change, contrary to our expectation, in EV consistency across the two payoff conditions between high and low numeracy groups. Lastly, we used moderation analysis to observe the effect of AED distribution on the relationship between payoff and adaptive strategy selection (embodied by the change in EV consistency). We identified that when AED remains constant, the change in payoff condition has no significant effect on EV consistency. However, change in AED has a significant direct effect on EV consistency.

This leads us to two conclusions. First, modulation in EV consistency is informed by understanding that the relative difference in value is significantly dissimilar across the two payoff conditions (i.e., EV ratio 1–2 in the low-payoff condition compared to EV ratio 5–6 in the high-payoff condition), but the magnitude of that effect is comparatively small. Second, participants did not recognize the relative difference in value, and the presence of two payoff conditions together has no significant effect on the changes in EV consistency. Instead, the change in EV consistency across the two payoff conditions is largely due to participants' response to the absolute difference in value (i.e., AED between options) regardless of the relative difference in value.

4. Experiment 2

In the second experiment, we plan to eliminate one of the two interpretations of the previous result. In Experiment 1, participants were presented with choice problems from both payoff conditions to provide them with the knowledge that the relative difference (i.e., EV ratio 1–2 in the low-payoff condition compared to EV ratio 5–6 in the high-payoff condition) in value across the two payoff conditions is significantly dissimilar. Hence, if the change in EV consistency is motivated by the relative difference in value, then the lack of information about the relative difference should, in principle, stop participants from making changes in their decision strategy (i.e., modulation in EV consistency between two payoff condition). We plan to test this hypothesis using the non-inferiority test. Put simply, Experiment 2 aims to test the non-inferiority of mean EV consistency in the low-payoff condition compared to mean EV consistency in the high-payoff condition.

If we find that there is no meaningful difference in EV consistency between the two-payoff conditions, then we can with sufficient confidence conclude that the change in EV consistency occurred due to participants' recognition of the relative difference in value between the two payoff conditions (i.e., EV ratio 1–2 in the low-payoff condition compared to EV ratio 5–6 in the high-payoff condition). Otherwise, the change in decision strategy across two the payoff conditions is due to participants' response to the absolute difference in value (i.e., AED between options).

4.1. Materials and procedure

We randomly presented each participant with 36 low payoff or 36 high-payoff choice problems. Unlike Experiment 1, participants were presented with either choice problems from the high-payoff or low-payoff condition, together with a numeracy scale. Presentation of choice problems and the numeracy scale was counter-balanced. Pre-registration, experimental procedure, data used for analysis, and complete analysis (R markdown file) of Experiment 2 have been posted on the Open Science Framework (https://osf.io/67hwc/).

4.1.1. Non-inferiority test

Employing the non-inferiority test enables us to prove that something does not exist (Streiner, 2003). In most instances, we design studies to show statistical significance using null-hypothesis significance testing (NHST)—that is, if we were to observe that p is less than or equal to 0.05, we would conclude that we can reject the null hypothesis and therefore accept the alternative—that there is a significant difference between the two groups. However, if p is greater than 0.05, we would not say that we can accept (or prove) the null hypothesis; rather, we would conclude that we failed to disprove the null. Therefore, such a methodology does not allow us to prove the nonexistence of something. In other words, using NHST, we cannot say that there is no practical difference between the two groups; hence we can reject the null hypothesis. In such cases, one uses the equivalence test to show that there is no meaningful difference between the means of two groups.

In the current study, we used the non-inferiority test paradigm (a specialized version of the equivalence test) to show that the mean of one group is neither meaningfully larger nor smaller than the other group. Importantly, rather than saying that the means of two groups have to be absolutely identical, we establish an acceptable range of closeness (δ) within which we would say that the groups are similar enough. Here, meaningfulness is not a statistical question, rather a practical one. Following the recommendation from Cribbie and Arpin-Cribbie (2009), we used a three-point scale, instead of a single absolute value, to establish the degrees of closeness (δ) between the two conditions (see, Appendix 4). If $\delta < 5\%$, then there is a "definitive non-inferiority" between the high-payoff and low-payoff condition. If $\delta < 10\%$, then there is a "probable non-inferiority" between the two conditions. Lastly, if $\delta < 15\%$, then there is an "anecdotal non-inferiority" present between the two conditions.

Therefore,

$$H_0 : \mu_0 - \mu_1 \geq \delta \tag{1}$$

$$H_1 : \mu_0 - \mu_1 < \delta \tag{2}$$

H_0 = Null hypothesis
H_1 = Alternative hypothesis
μ_0 = mean EV consistency in the high-payoff condition
μ_1 = mean EV consistency in the low-payoff condition
δ = acceptable difference

4.1.2. Objective statistical numeracy

We used the 3-item numeracy scale developed by Schwartz et al. (1997) and the BNT scale together to measure objective statistical numeracy and risk literacy. Possible scores ranged between 0 to 7 points, with higher scores indicating higher objective statistical numeracy.

4.1.3. Participants

The sample size was calculated following the formula provided by Chow, Shao, Wang, and Lokhnygina (2017) to obtain a power of 0.80 with a significance level (α) of 0.05 considering a non-inferiority margin of .08 (see, Appendix 3 for the exact formula). Two hundred and forty-eight volunteers (mean age = 27.7 years, SD = 8.92) participated in an online study for a half-hourly compensation of 2.5 GBP (equivalent to approximately 3.5 USD). Participants were recruited via the Prolific platform, where they were explicitly told that the current study would only examine their cognitive abilities (see, Appendix 11 for more detailed instructions). Furthermore, participants were told that they would be asked to solve a few tasks and questionnaires, and they would be compensated based on their performance in the task. As a bonus, in the low-payoff condition, participants were told that for every 1000 points, they would receive an additional 0.80 GBP on top of the flat fee. However, in the high-payoff condition, participants were told that for every 500 points, they would receive an additional 0.65 GBP on top of the flat fee. Lastly, participants were made aware that they could stop the task at any time. However, if they do so, they would not be able to continue the study further and would not receive any compensation. The difference in bonus payment between the two conditions is contingent upon the estimated earning difference between the two payoff conditions (See Appendix 5 for the exact calculation).

4.2. Results

4.2.1. Numeracy

The mean score of participants in the numeracy scale is 3.27 (SD = 2). Out of 248 participants, 111 participants are part of the high numeracy group (Numeracy $\geq$ 4), and 137 participants are part of the low numeracy group (Numeracy $\leq$ 3). Mean EV consistency in the high-payoff condition is higher for highly numerate individuals (M = 0.91, SD = 0.09) compared to less numerate individuals (M = 0.79, SD = 0.16). Similarly, mean EV consistency in the low-payoff condition is higher for highly numerate individuals (M = 0.83, SD = 0.11) compared to less numerate individuals (M = 0.73, SD = 0.13). Both results are consistent with the results from Experiment 1 (see Table 3).

Table 3
Descriptive table of individual measures.

	Numeracy	EV consistency		RT (log)	
		High payoff	Low payoff	High payoff	Low payoff
Median	3.000	0.861	0.806	8.020	8.140
Mean	3.262	0.838	0.781	8.036	8.128
SD	1.963	0.144	0.129	0.458	0.449
MAD	1.000	0.083	0.083	0.245	0.275
Variance	3.854	0.021	0.017	0.209	0.202

4.2.2. Choice problem

Mean EV consistency and RT in the high-payoff condition is 0.84 (SD = 0.14) and 8.04 (SD = 0.46), respectively. Whereas, the mean EV consistency and RT in the low-payoff condition is 0.78 (SD = 0.13) and 8.13 (SD = 0.45), respectively. We used the TOSTER R package to test the non-inferiority between the two payoff conditions using the independent groups Student's equivalence test[4] (Lakens, Scheel, & Isager, 2018). When $\delta < 5\%$ (definitive non-inferiority), the non-inferiority test was non-significant, $t(246) = 1.166$, $p = .878$, given the bounds of -Inf and 0.040 (on a raw scale) with an alpha of 0.05. Similarly, when $\delta < 10\%$ (probable non-inferiority), the non-inferiority test was non-significant, $t(246) = -1.166$, $p = .122$, given the bounds of -Inf and 0.080. However, when $\delta < 15\%$ (anecdotal non-inferiority), the non-inferiority test was significant, $t(246) = -3.497$, $p < .001$, given the bounds of -Inf and 0.120.

Looking at the mean EV consistency of both payoff conditions from Experiment 1 and Experiment 2, we can rule out the interpretation of anecdotal non-inferiority between the two payoff conditions due to the similarity of distribution and magnitude of modulation in decision strategy across the two payoff conditions.

Next, we conducted a moderation analysis and calculated the Johnson-Neyman interval. Results from the moderation analysis suggest that there is a significant interaction, $\beta = -0.034$, $Z = -8.19$, $p < .001$, between payoff and AED distribution on EV consistency. There is no significant direct effect, $\beta = 0.104$, $Z = 1.75$, $p = .08$, of payoff conditions on EV consistency. On the other hand, there is a significant direct effect, $\beta = 0.033$, $Z = 16.28$, $p < .001$, of AED on EV consistency. The current result mirrors the result from Experiment 1. Lastly, as Fig. 3B suggests, the Johnson-Neyman analysis indicates that when AED is outside the interval of the [23.18, 30.96] points, the slope of payoff difference is significant ($p < .05$). Put differently, when AED between options is between the [23.18, 30.96] points, there is no significant difference in EV consistency between the two payoff conditions. Unlike Experiment 1, we have a smaller interval of insignificance [23.18, 30.96].

4.3. Summary

Mean EV consistency is meaningfully higher in the high-payoff condition compared to the low-payoff condition. Put differently, mean EV consistency in the low-payoff condition is not non-inferior to the mean EV consistency in the high-payoff condition. The significant moderation effect and insignificant direct effect of payoff conditions on EV consistency argue that the nature of the relationship between the two payoff conditions and EV consistency is significantly moderated by absolute differences in values. Therefore, the result at hand conclusively indicates that the change in EV consistency is not motivated by the understanding of the relative difference in value (i.e., EV ratio 1–2 in low-payoff condition compared to EV ratio 5–6 in high-payoff condition). Thus, Experiment 2 successfully dissociated the relationship between the change in EV consistency and the payoff conditions. Lastly, we observed that the change in EV consistency across the two payoff conditions is, corroborating evidence from Experiment 1, participants' response to the absolute difference in value (i.e., AED between options).

5. Discussion

Earlier studies had delineated the importance of payoff conditions on adaptive decision making (Mondal, 2021; Traczyk, Sobkow, et al., 2018). However, the current results from both studies revealed that the payoff conditions alone do not invoke adaptive strategy selection, irrespective of numeracy levels. In earlier studies, multiple factors (i.e., lack of trade-off between strategies, asymmetry in AED distribution) were not consistent between the two payoff conditions. However, when controlled, we identified that numerate individuals are better at maximizing expected reward following absolute difference in value between options and not the relative difference in value embodied by two payoff conditions. Both experiments successfully dissociated the relationship between payoff conditions and changes in EV consistency.

The current result mirrors the result from context needed for numerically proficient individuals to make adaptive choices. Contexts such as lack of trade-off between strategies, asymmetry in AED distribution, and varying difficulty levels provide the information needed for highly numerate individuals to make more adaptive choices.
For example,

First sentence: Rahul entered the room and Raj started smiling.

[4] The results are consistent even if we conduct an equivalence test instead of the non-inferiority test.

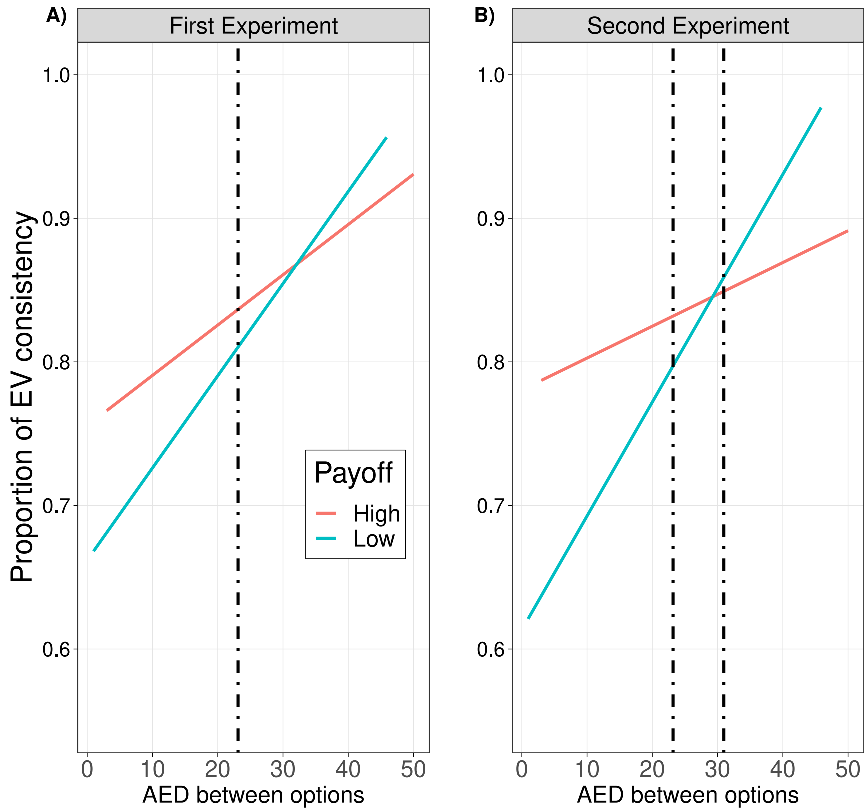

Fig. 3. EV consistency of Experiment 1 and Experiment 2 is plotted as a function of Absolute EV Difference (AED) across the two payoff conditions. Here, the dotted line refers to the Johnson-Neyman interval of insignificance. (A) In Experiment 1, once AED crosses the dotted line [23.11] there is no significant difference in EV consistency between the two payoff conditions. (B) In Experiment 2, there is no significant difference in EV consistency between the two payoff conditions in the interval between [23.18, 30.96].

Second sentence: Rahul entered the room and therefore Raj started smiling.

If someone asks us to explain what the first sentence means, we often presume that both events (i.e., entering the room and smiling) are related as described by the second sentence. This idea of causation comes from our proficiency in understanding implicit contexts (unlike other artificial systems). The context of a sentence plays a big part in the way we extract meaning from it (Conrad, 1974; Gigerenzer, 2007; Wlotko & Federmeier, 2012). Notably, age has a moderating effect on the ability to understand context information. As we grow older, our experience in the world helps us in decoding the context (from sarcasm to passive-aggressive anger) associated with sentences (Wlotko & Federmeier, 2012). Similarly, in earlier studies, multiple factors (i.e., lack of trade-off between strategies, asymmetry in AED distribution, and varying difficulty levels) more readily provided highly numerate individuals with the context they need to adaptively switch between optimal and sub-optimal strategies (i.e., make changes in EV maximization strategy) over the two payoff conditions compared to less numerate individuals. However, in the current study, when we controlled such factors to let just the EV ratio vary between two payoff conditions, participants did not adaptively modulate their decision strategy in accordance with the changes in payoff condition. Instead, they started maximizing EV following AED between options regardless of their numeracy levels.

Participants' judgment in both studies indicates that choices are made with respect to the frame of reference of AED distribution. Moderation analysis and the Johnson-Neyman interval of insignificance attest to the empirical validity of the frame of reference that led to the difference in strategies. The current result is in line with the prediction from Adaptation-Level Theory (ALT; Helson, 1964), one of the most prominent and widely used theory on stimulus frame of reference. The ALT argues that judgments are made in relation to adaptation level, and the adaptation level depends on all past and present stimuli. Put simply, the ALT postulates that people respond to the current stimulus using a frame of reference, which is a function of all earlier stimuli (Helson, 1948). Adaptation level can be defined as a region in the stimulus scale that produces indifferent responses. Results from the Johnson-Neyman interval of insignificance in both studies validate the notion of adaptation level and the region of indifferent responses. Support for adaptation-level theory is more prominent in the field of perceptual or psychophysical research for diverse phenomena such as constancy, contrast, and adaptation in the domain of vision, hearing, smell, taste, etc. Bevan, Pritchard, and Reed (1962), Helson (1947), Hulshoff Pol, Hijman, Baaré, and van Ree (1998). Nevertheless, results from the current study indicate that adaptation level can also be found in the cognitive domain (i.e., adaptation to value distribution) as well. Lastly, future studies should investigate whether the range of indifference found in the current study remains the same with different AED levels or changes with a specific AED distribution.

The current study also highlights the importance of focusing on absolute values. Almost all modern theories of decision making are built on the idea of relativity (Friedman & Savage, 1952; Tversky & Kahneman, 1992), where preference for an item is judged with respect to other items. The higher value object is consistently preferred if the difference in value is relatively large between items. Although, as the psychological interpretations of the law of diminishing returns dictate, preferences become more inconsistent when the difference in value is relatively similar (Shevlin, Smith, Hausfeld, & Krajbich, 2022). However, the current result is inconsistent (in agreement with Shevlin et al., 2022) with the hypothesis of diminishing value sensitivity. We have identified a more complex relationship between relativity and preference consistency. As noted in the previous section, we argued that the nature of the relationship between consistency of preference and the relative difference in value is anchored to the absolute difference in value one expects to earn or lose. For example, when the absolute difference between two offers is 50 $ and the relative difference is large (i.e., option A is 10$ and option B is 60$), preference for the higher value item is consistent and in accordance with the hypothesis of diminishing value sensitivity. Similarly, when the absolute difference between two offers is 50 $ and the relative difference is small (i.e., option A is 400$ and option B is 450$), preference for the higher value item is relatively less consistent and in accordance with the hypothesis of diminishing value sensitivity. Following the same logic, when the absolute difference between two offers is 200 $ and the relative difference is large (i.e., option A is 50$ and option B is 250$), preference for the higher value item is consistent and in accordance with the hypothesis of diminishing value sensitivity. However, the current study revealed that when the absolute difference between two offers (i.e., option A is 1000$ and option B is 1200$) is large (i.e., 200 instead of 50) and the relative difference is small, preference for the higher value item is equally as consistent as it was when relative difference was large and against the prediction of the hypothesis of diminishing value sensitivity. We identified a boundary condition for the hypothesis of diminishing value sensitivity. We speculate that the boundary condition is modulated by risk appetite, where individuals with more risk appetite will be more inconsistent compared to individuals with less risk appetite. However, to say anything conclusively, more work needs to be done in the matter.

Classical decision theory assumes that people make rational choices and prefer options that maximize subjective expected utility. However, human decisions can be driven by various motives. For example, it has been repeatedly demonstrated that some people prefer the best option (i.e., maximizers), but others prefer the good-enough option (i.e., satisfies) (Schwartz et al., 2002). In line with this distinction, decision-makers may exhibit different sensitivity to changes in EVs, depending on their goal (whether to maximize or satisfy).

The present findings can be interpreted in the light of personal goals set by participants and their sensitivity to EV changes. That is, previous studies on numeracy and adaptive strategy selection (Traczyk, Sobkow, et al., 2018) employed two payoff conditions that included either meaningful (i.e., maximizing EV could lead to a substantially greater payoff) or trivial (i.e., making random choice led to comparable payoffs) choice problems. Consequently, participants with high numeracy who were more sensitive to EV could adapt their behavior by putting more effort into solving meaningful choice problems and, at the same time, they employed a less effortful heuristic strategy or made a fast random choice in trivial problems. In other words, depending on the structure of the choice task, they modified their personal goals to maximize the payoff or make a satisfying choice that minimized their effort and saved time. The current study with evenly distributed choice problems eliminated the opportunity to easily compare the potential payoff and understand the importance of choice problems. Consequently, participants were not able to manage their time and effort, so this goal was eclipsed by the goal of maximizing EV.

Theoretical models (Lopes, 1987) and empirical research (Wulff, Hills, & Hertwig, 2015) have already focused on the aspiration level and tested how instructions to follow a particular decision strategy (Schoemann, Schulte-Mecklenbeck, Renkewitz, & Scherbaum, 2019) and achieve a specific goal (Jarecki & Rieskamp, 2020) shape decision-making process. We believe that introducing the concept of personal goals to decision-making research can provide a promising new avenue for future research. It would be especially helpful in studying adaptive strategy selection since rational behavior in dynamic and complex environments might be evidenced by choices that increase the likelihood of achieving a personal goal (Baron, 2008).

Finally, it is worth noting that the present study does suffer from some limitations. Consistent with earlier studies (Mondal, 2021; Pachur et al., 2013; Traczyk, Sobkow, et al., 2018), the current study also suffers from varying difficulty levels across two payoff conditions. Future work should investigate whether there is a relationship between adaptive strategy selection and varying difficulty levels across two payoff conditions. Second, all previous studies and the current study focused exclusively on monetary gambles to test adaptive strategy selection. However, in everyday life, one encounters more value-based situations than monetary problems. Also, value-based problems would help us distinguish between true preference and mistakes made while expressing that preference across varied numerically able participants (Drouvelis et al., 2020; Lilleholt, 2019; Mechera-Ostrovsky, Heinke, Andraszewicz, & Rieskamp, 2022). Lastly, the current study used computational models to explore changes in decision strategy. Future studies can use process tracking methods (such as retrospective verbalization) to evaluate the actual decision strategy employed by participants.

In summary, we illustrated that individuals do not make adaptive strategy selection in the presence of two payoff conditions alone, regardless of the numeracy level of the individuals. Instead, the change in EV consistency across two payoff conditions is primarily due to participants' responses to the absolute difference in value. We successfully dissociated the relationship between EV consistency and payoff conditions. Lastly, we highlighted the anchoring effect of absolute difference in value in human preference and choice consistency. In conclusion, we demonstrated that numerate individuals, compared to less numerate individuals, maximized their expected reward more consistently following absolute differences in values irrespective of the relative differences in value provided by the two payoff conditions.

Data availability

Pre-registration, experimental procedure, choice problems, data, and complete analysis has been posted on the OSF repository. Data are available at OSF: Experiment 1 (https://osf.io/p8av4/) & Experiment 2 (https://osf.io/67hwc/).

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.joep.2023.102611.

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10.2.1 Supplementary Materials

Conditionality of adaptiveness: Investigating the relationship between numeracy and adaptive behavior

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Appendix

1 EV calculation

$$EV = \Sigma(p_i O_i) \tag{1}$$

where, p and O are probabilities and outcomes, respectively, associated with each possible outcome $(i_1, . . . , n)$ of that option.

1.1 EV ratio

$$\frac{Max(EV_i, EV_j)}{Min(EV_i, EV_j)} \tag{2}$$

1.2 Absolute EV difference (AED) calculation

$$AED = \mid EV_i - EV_j \mid \tag{3}$$

i and j captures option A and option B, respectively.

2 Modulation in decision strategy

$$\mu_{md} = \mu_{hp} - \mu_{lp} \tag{4}$$

μ_{md} = Modulation in decision strategy

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μ_{hp} = Mean EV consistency in high-payoff condition
μ_{lp} = Mean EV consistency in low-payoff condition

3 Sample size calculation for the Experiment 2

$$n = \left(\delta \frac{Z_1 - \alpha + Z_1 - \beta}{\mu - \mu_0 - \delta}\right)^2 \tag{5}$$

$$1 - \beta = \Phi(Z - Z_1 - \alpha) + \phi(-Z - Z_1 - \alpha) \tag{6}$$

where,

$$Z = \frac{\mu - \mu_0 - \delta}{\sigma/\sqrt{n}} \tag{7}$$

n= sample size
σ = standard deviation
ϕ = standard Normal distribution function
α = Type I error
β = Type II error, meaning power = 1- β
σ = Non-inferiority margin

4 Non-inferiority test

In the non-inferiority test, the range of acceptable difference (δ) is calculated following the mean EV consistency (M = 0.81) of Experiment 2.
For definitive non-inferiority ($\delta < 5\%$) test

$$5\% \times .81 \approx .04 \tag{8}$$

For probable non-inferiority ($\delta < 10\%$) test

$$10\% \times .81 \approx .08 \tag{9}$$

For anecdotal non-inferiority ($\delta < 15\%$) test

$$15\% \times .81 \approx .12 \tag{10}$$

5 Bonus payment calculation for Experiment 2

In Experiment 1, we paid 1 GBP per 1000 points. If participants make EV consistent choices, we calculated[1] that they would earn approximately 3.2 GBP (mean of EV consistency is approximately 3154.5 points). In order to control the effect of bonus payment on preference,

[1]We simulated outcomes 1000 times and took the mean value.

we decided to give equal bonuses in the high- and low-payoff condition. Therefore, given that participants' expected earning is 3.2 GBP, we will pay 1.6 GBP per payoff condition.

However, due to the AED control, participants have a higher chance of accumulating more points in low-payoff conditions than high-payoff conditions. As a result, we divided 1.6 GBP based on reward earned following EV consistent choice.

Thereby, we paid .08 GBP per 100 (.08*20 = 1.6 GBP) points to participants in the low-payoff condition considering the mean reward earned following EV maximization strategy (approximately 2001.9 points). On the other hand, we paid .13 GBP per 100 (.13*12 = 1.56 GBP) points to participants in the high-payoff condition considering the mean reward earned following EV maximization strategy (approximately 1152.6 points).

6 Controlled variables

We tested whether variance, outcome, probability, and expected value of option A differ significantly from option B over 72 choice problems. We did not find any significant differences in these dimensions, suggesting that the only factor varying between the options was the EV ratio.

Table 1: Paired Samples T-Test

Measure 1		Measure 2	t	df	p	Cohen's d
Variance A	-	Variance B	0.247	71	0.805	0.029
Probability A	-	Probability B	-1.206	71	0.232	-0.142
Outcome A	-	Outcome B	0.076	71	0.940	0.009
Expected Value A	-	Expected Value B	-1.244	71	0.218	-0.147

6.1 Assumption Check

Table 2: Shapiro-Wilk test of Normality

			W	p
Variance A	-	Variance B	0.988	0.728
Probability A	-	Probability B	0.986	0.629
Outcome A	-	Outcome B	0.989	0.776
Expected Value A	-	Expected Value B	0.960	0.021

6.2 Descriptive Table

TABLE 3: Descriptive statistics

	N	Mean	SD	SE
Variance A	72	998.281	755.845	89.077
Variance B	72	971.538	716.706	84.465
Probability A	72	0.532	0.277	0.033
Probability B	72	0.587	0.294	0.035
Outcome A	72	56.708	28.076	3.309
Outcome B	72	56.403	26.736	3.151
Expected Value A	72	29.922	23.680	2.791
Expected Value B	72	33.950	25.560	3.012

7 Controlled variables across two-payoff condition

We tested whether the difference between options, across the two payoff conditions, differ in case of variance, outcome, and probability over the choices problems. We did not find any significant differences in these dimensions, attesting that the only factor varying over two payoff conditions was the EV ratio.

TABLE 4: Independent Samples T-Test

	t	df	p	Hedges' g
Difference in Variance	-0.094	70	0.925	-0.022
Difference in Outcome	0.000	70	1.000	0.000
Difference in Probability	-0.358	70	0.722	-0.083

7.1 Assumption Check

Table 5: Shapiro-Wilk test of Normality

	Condition	W	p
Difference in Variance	High	0.985	0.889
	Low	0.974	0.534
Difference in Outcome	High	0.982	0.803
	Low	0.977	0.645
Difference in Probability	High	0.961	0.224
	Low	0.979	0.709

7.2 Descriptive Table

Table 6: Group Descriptives

	Condition	N	Mean	SD	SE
Difference in Variance	High	36	16.451	923.015	153.836
	Low	36	37.035	925.226	154.204
Difference in Outcome	High	36	0.306	39.559	6.593
	Low	36	0.306	28.209	4.702
Difference in Probability	High	36	-0.073	0.477	0.079
	Low	36	-0.039	0.292	0.049

8 Model comparison

We simulated four prominent models of decision making to observe the similarity in predictions across 72 choice problems. As Figure 1 suggests, the prediction is similar between the Expected Value maximization model (EV), Cumulative Prospect Theory (CPT), and Expected Utility Theory (EU). In percentage terms, there is no difference (i.e., both prediction matches 100% of the time) in prediction between the EV maximization model and CPT with standard parameters (taken from the study by Tversky Kahneman, 1992) for the current set of choice problems. Similarly, there is a close match (i.e., 97.2%, 70 out of 72 problems) in the prediction of the standard EU (when α = .8, γ = 2) model with isoelastic power utility function and EV maximization model. Even for the lowest matching models, between EV and Priority heuristics (PH) model, the similarity is 83.3% (or 60 out of 72 problems). Similar approaches were also used in earlier studies (Pachur et al., 2013; Traczyk et al., 2018). To summarise, with the current set of choice problems, the predictions made by various models are very similar.

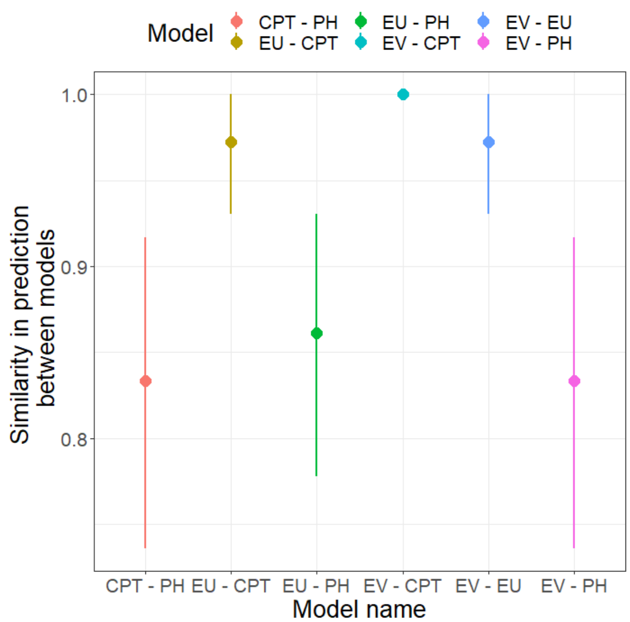

FIGURE 1: Similarly between different models in being plotted. Here, EU-PH refers to comparison between Expected utility (EU) and Priority Heuristics (PH) model. EU-PT refers to comparison between EU and Cumulative Prospect Theory (CPT). EV-EU refers to comparison between Expected Value (EV) maximization and EU model. EV-PH refers to comparison between EV and PH model. EV-PT refers to comparison between EV and PT model. Lastly, PT-PH refers to comparison between PT and PH model.

9 Change in strategy over two-payoff condition

1. The high-payoff condition represents decision strategy consistent with the prediction from the Cumulative Prospect Theory (CPT)/Expected value maximization model (EV). As shown in Figure 1, predictions from CPT and EV models are the same.
2. The low-payoff condition represents decision strategy consistent with the prediction from the Priority Heuristics (PH).

9.1 Experiment 1

Similar to earlier method of analysis, the current analysis also suggest that there is a significant difference in strategy between the two payoff conditions. However, that difference in strategy does not constitute adaptive change in decision strategy.

TABLE 7: Paired Samples T-Test

Condition		Condition	W	p	Rank-Biserial Correlation
High-payoff	-	Low-payoff	2820.500	< .001	0.699

TABLE 8: Shapiro-Wilk test of Normality

	W	p
High-payoff - Low-payoff	0.973	0.043

TABLE 9: Descriptive Statistics

	N	Mean	SD	SE
High-payoff	95	0.846	0.135	0.014
Low-payoff	95	0.789	0.128	0.013

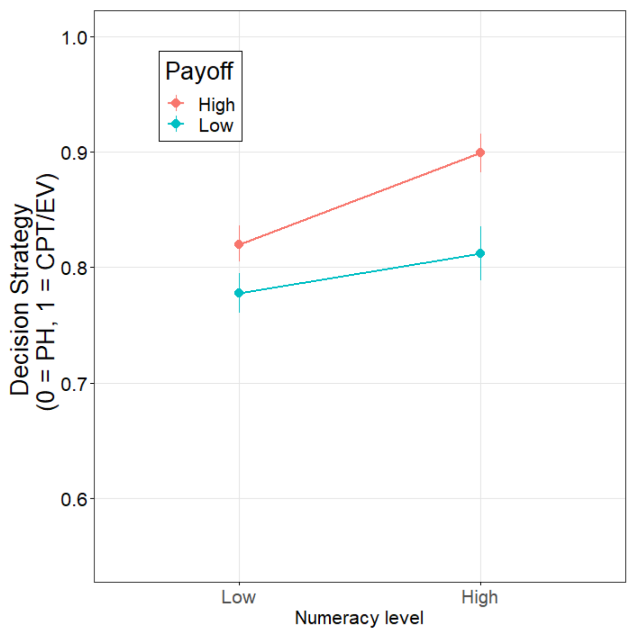

FIGURE 2: Decision strategy as a function of Numeracy level under varied payoff condition. Here, 0 = PH, refers to choices consistent with Priority heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.

9.2 Experiment 2

Similar to Experiment 1, the current analysis also suggests that there is a significant difference in strategy between the two payoff conditions. In other words, even though participants did not have information regarding the relative difference between the two payoff conditions, there is still significant difference in strategy over the two payoff conditions, attesting to the fact that the EV ratio is not responsible for changes in strategy over the two payoff conditions.

TABLE 10: Mann-Whitney test

	W	p	Rank-Biserial Correlation
Modulation in strategy	9686.000	$< .001$	0.260

TABLE 11: Shapiro-Wilk test of Normality

		W	p
Modulation in strategy	High	0.882	$< .001$
	Low	0.877	$< .001$

TABLE 12: Group Descriptives

	Condition	N	Mean	SD	SE
Modulation in strategy	High	124	0.838	0.144	0.013
	Low	124	0.791	0.129	0.012

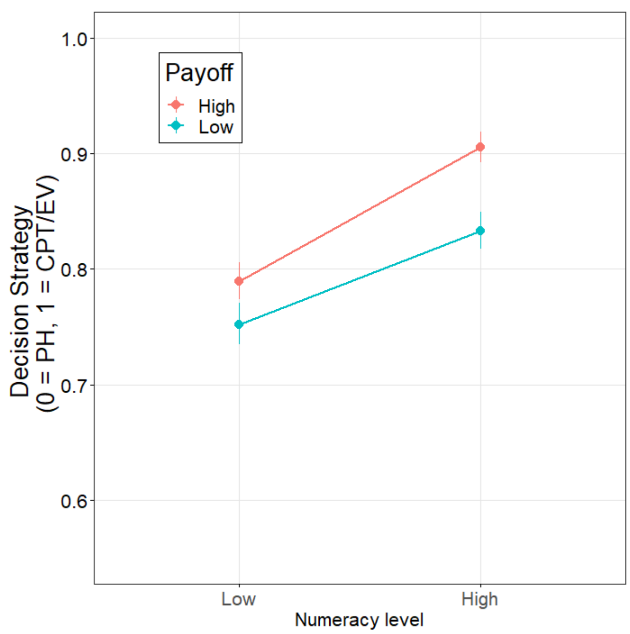

FIGURE 3: Decision strategy as a function of Numeracy level under varied payoff condition. Here, 0 = PH, refers to choices consistent with Priority heuristic; 1 = CPT/EV refers to choices consistent with Cumulative Prospect Theory/Expected Value.

10 Instruction for Experiment 1

Inquisit files of Experiment 1 (Experiment 1 zip) are provides in the OSF repository.

10.1 General instruction

The aim of the study is to measure cognitive abilities of people belonging to different social and demographic groups. On the following pages, we will ask you to solve a few tasks and questionnaires. You will be compensated based on your performance in those tasks. Depending on your pace of work, the entire study will take around 20 minutes to finish. It is very important during this time to be fully dedicated to answering the questions that appear on the screen.

Therefore, we ask:

1. Give answers yourself (do not consult with other people).
2. Turn off any sound sources (e.g. music or TV).
3. Keep a piece of paper and a pen beside you, it may be useful in some tasks.
4. Answer honestly-only honest answers count for us.
5. Reserve a sufficient amount of time to participate in the study (if you do not have time now, start the study later at some convenient time).

READ INSTRUCTIONS CAREFULLY BEFORE EACH TASK

- You can stop the task at any time by pressing 'CTRL + Q' keys together. However, if you do so, you will not be able to continue the study further and won't receive any compensation.
- If you are ready to continue, press the 'Next' button. To stop the test, press 'CTRL+Q'.

Good luck :)

10.2 Specific instruction for choice problems

In this task, we will present you with some gambles where each gamble will have two options. For example:

A) 55% chance to receive 11 point B) 4% chance to receive 68 points

If you select the option on the left side (by clicking on it), you will have a 55% chance of winning 11 points and 45% probability of getting 0 points. However, if you choose the option on the right side (by clicking on it), you will have a 4% chance of winning 68 points and 96% probability of getting 0 points.

The more points you will collect during the task, the more money you can earn after finishing the study. So, for every 1000 points, you will receive an additional 1 pound on top of the flat fee you have already received. So if you in total collect 3000 points, then in total you will earn 4.67 pounds. Hence, familiarize yourself with each gamble and choose the one that is more attractive to you.

If you are ready to start the task, click Next.

10.3 Specific instruction for Numeracy task

On the following pages, you will be asked to solve few problems.

1. Do not use a calculator but feel free to use paper for notes.
2. You can take as long as you want to solve each problem.

When you are ready to start, please press the 'Next' button.

11 Instruction for Experiment 2

Inquisit files of Experiment 2 (Experiment 2 zip) are provides in the OSF repository.

11.1 General instruction

The aim of the study is to measure cognitive abilities of people belonging to different social and demographic groups. On the following pages, we will ask you to solve a few tasks and questionnaires. You will be compensated based on your performance in those tasks. Depending on your pace of work, the entire study will take around 20 minutes to finish. It is very important during this time to be fully dedicated to answering the questions that appear on the screen.

Therefore, we ask:

1. Give answers yourself (do not consult with other people).
2. Turn off any sound sources (e.g. music or TV).
3. Keep a piece of paper and a pen beside you, it may be useful in some tasks.
4. Answer honestly-only honest answers count for us.
5. Reserve a sufficient amount of time to participate in the study (if you do not have time now, start the study later at some convenient time).

READ INSTRUCTIONS CAREFULLY BEFORE EACH TASK

- You can stop the task at any time by pressing 'CTRL + Q' keys together. However, if you do so, you will not be able to continue the study further and won't receive any compensation.
- If you are ready to continue, press the 'Next' button. To stop the test, press 'CTRL+Q'.

Good luck :)

11.2 Specific instruction for participants in high-payoff condition

In this task, we will present you with some gambles where each gamble will have two options. For example:

A) 55% chance to receive 11 point B) 4% chance to receive 68 points

If you select the option on the left side (by clicking on it), you will have a 55% chance of winning 11 points and 45% probability of getting 0 points. However, if you choose the option on the right side (by clicking on it), you will have a 4% chance of winning 68 points and 96% probability of getting 0 points.
The more points you will collect during the task, the more money you can earn after finishing the study. So, for every 500 points, you will receive an additional 0.65 pound on top of the flat fee you have already received. So if you collect 1500 points, then in total you will earn 2.79 (1.95+0.84) pounds. Hence, familiarize yourself with each gamble and choose the one that is more attractive to you.

If you are ready to start the task, click Next.

11.3 Specific instruction for participants in low-payoff condition

In this task, we will present you with some gambles where each gamble will have two options. For example:

A) 55% chance to receive 11 point B) 4% chance to receive 68 points

If you select the option on the left side (by clicking on it), you will have a 55% chance of winning 11 points and 45% probability of getting 0 points. However, if you choose the option on the right side (by clicking on it), you will have a 4% chance of winning 68 points and 96% probability of getting 0 points.
The more points you will collect during the task, the more money you can earn after finishing the study. So if you in total collect 3000 points, then in total you will earn 3.24 (2.40+0.84) pounds. Hence, familiarize yourself with each gamble and choose the one that is more attractive to you.

If you are ready to start the task, click Next.

11.4 Specific instruction for Numeracy task

On the following pages, you will be asked to solve few problems.

1. Do not use a calculator but feel free to use paper for notes.
2. You can take as long as you want to solve each problem.

When you are ready to start, please press the 'Next' button.

12 Reference

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10.3 Third Published Manuscript

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Recurring Suboptimal Choices Result in Superior Decision Making

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OPEN DATA OPEN MATERIALS PREREGISTERED

A vast body of research has indicated that intensified deliberation on choice problems often improves decision accuracy, as evidenced by choices that maximize expected value (EV). However, such extensive deliberation is not always feasible due to cognitive and environmental constraints. In one simulation study and three well-powered fully incentivized empirical studies, using the decision-from-experience task, we found that individuals who maximized EV without time constraints accumulated higher total gain. The trend reversed in the following two studies. Under time constraints, participants who made more suboptimal (or random in terms of EV maximization) decisions earned more money than those who spent more time maximizing EV. By comparing sampling and decision strategies among people with higher and lower statistical numeracy, we found that more numerate individuals made quicker suboptimal choices, resulting in better overall earnings than less numerate individuals. Detailed analysis indicated that skilled decision makers sampled information more rapidly and dynamically. They adaptively relied on varying search strategies, initially focusing on reducing uncertainty and later discovering unobserved outcomes. Finally, adaptive exploration was accompanied by the development of a metacognitive understanding of the task structure and choice environment. Participants who recognized the effectiveness of the random selection strategy earned more rewards. Taken together, these findings suggest that people (especially those with higher numeracy) in time-constrained environment adaptively changed their decision-making strategies and developed a metacognitive understanding of the task structure and decision environment. This resulted in making recurring suboptimal choices that led to superior long-term performance in the decision task.

Keywords: numeracy, optimal decisions, adaptive decision making, expected value

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Supratik Mondal played a lead role in conceptualization, formal analysis, investigation, methodology, project administration, visualization, supervision, writing–original draft, and writing–review and editing. Domink Lenda played a lead role in funding acquisition, conceptualization, investigation, methodology, project administration, writing–original draft, and writing–review and editing. Jakub Traczyk played a lead role in funding acquisition, conceptualization, formal analysis, investigation, methodology, project administration, visualization, supervision, writing–original draft, and writing–review and editing.

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The preregistered design and analysis plan is accessible at https://doi.org/10.17605/OSF.IO/K7TFX and https://doi.org/10.17605/OSF.IO/U59FC.

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Empirical results and theoretical models in decision making suggest that intensified deliberation on choice problems often improves decision accuracy (e.g., selecting an option that maximizes expected value or utility). However, due to cognitive (e.g., numerical abilities of a decision maker) and environmental (e.g., time limits) constraints, deliberation is not always feasible, and individuals must employ different choice strategies to improve their decision quality (e.g., fast heuristic strategies; Gigerenzer & Goldstein, 1996; Gigerenzer & Selten, 2002; Simon, 1990a). In order to arrive at superior decisions in a time-constrained environment, individuals must balance the computational and temporal costs of making accurate choices to increase their chance of resource-rational behavior (Lieder & Griffiths, 2020).

Examples of cognitive limitation and environmental constraints dictating human decision making can be regularly witnessed in various walks of life. For instance, finding the best apartment or parking spot can be time consuming and cognitively effortful, especially when a decision maker has to sequentially and iteratively acquire and use information. In a city like London, where rental demand is high, engaging in an extensive property search can incur significant costs. One may need to decide between settling for a "good enough" property or continuing the search for a more suitable place. However, this prolonged search could result in losing the current "good enough" apartment without finding a better alternative. In such dynamic situations, brute-force maximization in a constrained environment may lead to inferior decisions. In contrast, recurring suboptimal choices (i.e., fast suboptimal decisions made in sequential choice problems) in the abovementioned settings can result in better performance in the long run. That is, people who comprehend the structure of the decision task can trade-off the decision accuracy and computational cost by making numerous faster but not optimal choices (i.e., settling for a good enough apartment whenever searching for a new apartment) will save their time and cognitive resources without incurring severe opportunity costs.

In the present study, we empirically test this specific pattern of behavior. Specifically, we examine whether decision makers are able to better "see" the structure of the task and adapt their decision strategy in accordance with the constraints associated with it (Dawes, 1979). We hypothesize that in an unconstrained choice environment, participants will employ more deliberative processes to achieve higher total rewards. However, under time constraints, some individuals (especially skilled decision makers with higher statistical numeracy) will make recurring suboptimal choices by adapting their exploration strategy to encounter more choice problems and earn higher rewards (in the current series of studies, we define the suboptimal choice as selecting a gamble with a lower expected value, EV). In other words, making faster recurring suboptimal (or random in terms of EV maximization) decisions may result in an overall superior performance (e.g., earning more money). We demonstrate that this effect is more pronounced among individuals with higher statistical numeracy. It manifests as changes in their decision strategy, exploration behavior, and enhanced metacognitive understanding of the choice task. These findings are consistent across various sets of choice problems and remain robust when controlling for fluid intelligence.

Theoretical Overview

Adaptive Exploration

Real-world situations are full of uncertainty, complexity, and constraints that, along with the limited computational capabilities of decision makers, often impede the application of an exhaustive, brute-force optimization process (Simon, 1990b). In such situations, suboptimal strategies are likely to approximate optimal strategies to a satisfactory level or sometimes even outperform them (DeMiguel et al., 2009; Gigerenzer, 2008).

It is well-established that individuals can adapt (i.e., balance between effort and accuracy) their decision-making strategies in response to the changes in task demands (Payne & Bettman, 2004; Payne et al., 1993). For example, the effectiveness of normative models (like EV, often used as a standard for accuracy) was found to be comparable to, or even less effective than, heuristic or random strategies under severe time pressure (Payne et al., 1996). When faced with time pressure or high opportunity costs, decision makers may be unable to perform all the necessary processing operations, leading to a decline in performance when relying solely on the EV maximization strategy. To overcome this limitation, individuals can increase their information processing speed to explore and assimilate more information in a shorter period. However, it is not often feasible in the real world. Alternatively, they can shift their focus from the depth of evaluation to the breadth of evaluation (Payne et al., 1996) to construct a meaningful representation of a decision problem. These processing strategies can be examined by studying how individuals explore the structure of choice problems within a sampling paradigm (Hertwig et al., 2004; Wulff et al., 2018).

In the sampling paradigm, people can freely explore a distribution of payoffs to learn about outcomes and their respective probabilities. Crucially, individuals (based on their understanding of the structure of the choice problem) can determine the necessary amount of information required to stop sampling and indicate a response. Numerous studies have shown that people often rely on small samples when making decisions. This approach might be seen as suboptimal because it reduces the likelihood of observing rare events and creates an inaccurate representation of choice problems, which may hinder decision accuracy, such as those aimed at maximizing EV (Frey et al., 2014; Hertwig et al., 2004; Wulff et al., 2018).

However, exploration based on a limited number of samples is often adaptive for several reasons. First, it conserves cognitive resources, as decision makers do not need to store extensive sampling information in their working memory (Rakow et al., 2008). Second, small sample sizes can amplify the differences between experienced payoffs. This amplification effect makes options appear more distinct and makes choices easier for individuals (Hertwig & Pleskac, 2010). Finally, small samples help balance the time invested in exploring payoff distributions with the value of the information gained. In essence, while larger samples provide a more accurate representation of underlying payoff distributions, the incremental value of newly sampled information decreases over time as more samples are collected. For instance, after drawing just one sample, Hertwig and Pleskac (2008) demonstrated that the probability of choosing an option with a higher EV was approximately 60%. Drawing an additional five samples increased accuracy by 18%, while further increments in the number of draws—from five to 10 samples and from 10 to 20 samples—improved accuracy by 6% and 4%, respectively.

In time-constrained decision-making scenarios, the optimal strategy to maximize expected returns might be not to draw any sample at all (Ostwald et al., 2015). For instance, Vul et al. (2014), in their analysis of sampling in a two-alternative forced-choice task, found that when the temporal cost of acquiring new information is high, making multiple quick, albeit locally suboptimal, decisions based on very few samples can emerge as the globally optimal strategy in the long run. For instance, an individual is faced with choosing between two gambles, H and L, where the EV of H is greater than L, and the EV ratio[1] is 1.25. If the person repeatedly selects gamble H with a probability (q) of 0.8 and gamble L with a probability of $(1 - q) = 0.2$, then to receive the same reward in the long run, the person would need to choose gamble L five times more often than gamble H (see, Figure 1).

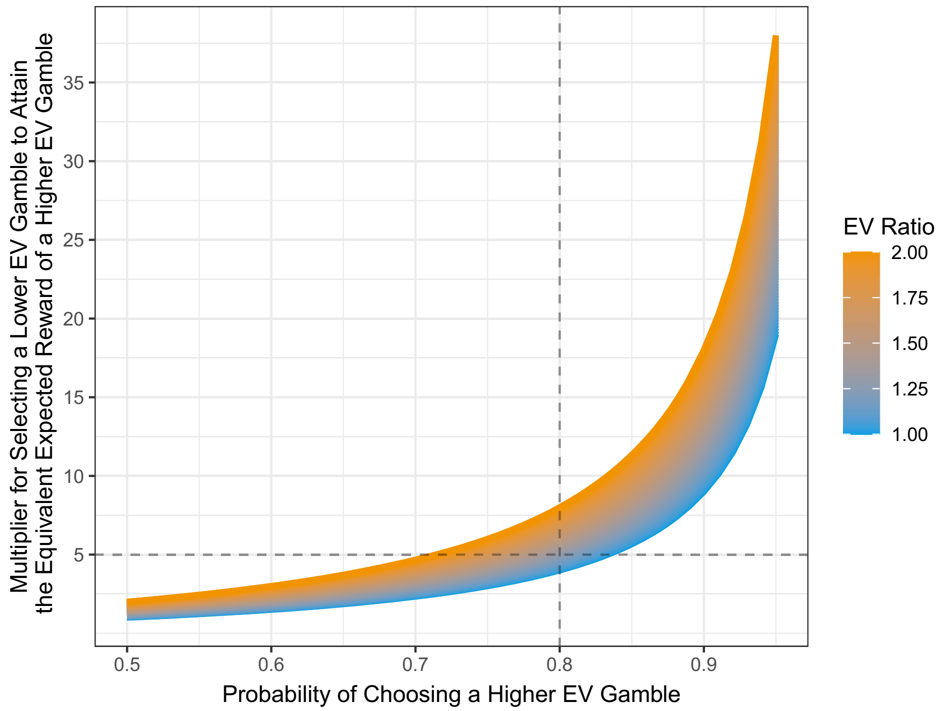
In a sampling paradigm, to achieve this level of reward within a fixed period, a decision maker can increase their sampling speed or decrease the number of samples. They can also employ different sampling strategies when making decisions to balance the costs and benefits of information search dynamically. Recent empirical evidence suggested (Spektor & Wulff, 2023) that decision makers can rely on distinct search strategies. For instance, the uncertainty-driven search focuses on options with higher variance in outcomes, using more samples to reduce uncertainty and improve decision quality. The value-driven search allocates more samples to options with a higher experienced mean, aiming to maximize subjective value based on past performance. Finally, the discovery-driven search concentrates on exploring unknown aspects of the choice problem by adaptively allocating samples to options suspected of containing unobserved outcomes, aiming to uncover new, crucial information.

Adaptive reliance on search strategies may facilitate the development of metacognitive understanding, enabling individuals to avoid repeating mistakes and efficiently allocate time and resources to necessary decision problems (Yeung & Summerfield, 2012). In other words, people become more sensitive not only to the strength of evidence (e.g., the mean of experienced outcomes), but also to the weight of evidence (e.g., the reliability of outcomes inferred from the variance of outcomes with sample size as a proxy; Griffin & Tversky, 1992). For example, an individual using a discovery-driven search, in a

[1] EV ratio $= \frac{EV_{\text{Gamble H}}}{EV_{\text{Gamble L}}}$

Figure 1

The Figure Quantifies How Much the Lower EV Gamble's Reward Needs to Be Multiplied to Match the Expected Reward of the Higher EV Gamble Based on Varying Probabilities and EV Ratios



Note. Dashed lines illustrate an example presented in the main text. EV = expected value. See the online article for the color version of this figure.

series of choice problems, may come to understand the structure of the choice environment at the beginning of the decision task by combining their experienced outcomes and variance. This understanding might lead to specific search patterns and choices that boost the likelihood of achieving long-term goals, such as incrementally increasing earnings across multiple trials rather than focusing on a single trial, resulting in *good decisions*.

Good Decisions and Skilled Decision Makers

The idea of good decision-making stems from the formulation of neoclassical normative theories. Normative decision theories serve as a standard for optimal human decision making (Thaler, 2018). Formally, the EV model (i.e., a sum of future outcomes multiplied by the probability of their occurrence) is a benchmark for making optimal choices under risk (assuming that a decision maker aims to maximize their rewards). However, since the St. Petersburg paradox described by Bernoulli (Bernoulli, 1954), the EV maximization principle has been challenged as a valid positive model. Alternative explanations of human decision making have instead considered the maximization of expected utility (Von Neumann & Morgenstern, 1944), maximization of subjective value (Tversky & Kahneman, 1992), satisfying (Simon, 1990b), aspirations (Lopes, 1987), or feelings (Loewenstein et al., 2001) as pivotal factors that motivate choices and shape the human decision-making process. Despite the irrefutable advantage of these descriptive models for theorizing about human decision making, from a formal perspective, the EV model still functions as a reference point for making optimal choices under risk. In addition, the EV model allows us to measure the change in decision strategy (i.e., maximization) corresponding to changes in the task structure.

A vast body of evidence suggests that skilled and numerate decision makers, who understand and utilize the concepts of probability and statistics, are more likely to make better decisions than individuals who are less statistically numerate (Cokely et al., 2018; Garcia-Retamero et al., 2019;

Peters, 2012; Reyna & Brust-Renck, 2020). Consistent with these findings, in a longitudinal study on a large sample of Dutch adults, Estrada-Mejia et al. (2016) found that numeracy is a crucial determinant of wealth accumulation trajectories over time. While participants with low numeracy tended to experience a decrease in wealth, those with high numeracy maintained a constant level of wealth. Authors estimated that a 1-point growth in numeracy score, on average, is linked with a 5% increment in personal wealth.

Among possible reasons for a greater wealth accumulation by highly numerate individuals, research has consistently indicated that there is a strong positive relationship between numeracy and choices maximizing EV (Cokely & Kelley, 2009; Millroth & Juslin, 2015; Mondal, 2021; Sobkow et al., 2020). In addition, highly numerate individuals are more sensitive to variations in EVs than less numerate individuals (Jasper et al., 2013; Peters & Bjalkebring, 2015).

Notably, prior findings have indicated that individuals with higher numeracy can follow the normative strategy of maximizing EV and modify their decision strategy based on the task's characteristics (Traczyk, Sobkow, et al., 2018). For instance, individuals with higher statistical numeracy, compared to those with lower statistical numeracy, maximized EV and made choices consistent with the predictions of the cumulative prospect theory (CPT; Tversky & Kahneman, 1992) when faced with meaningful choice problems (i.e., the relative difference in EVs between gambles was high). Conversely, in trivial problems (i.e., where the relative difference in EVs between gambles was low and the potential outcomes were comparable), more numerate individuals did not maximize EV and made choices consistent with the predictions of the priority heuristic (PH; Brandstätter et al., 2006). It suggests that individuals with higher numeracy comprehended the underlying payoff structure of the task and employed a more effortful and time-consuming compensatory decision strategy in meaningful decision problems, while for trivial problems, they adopted fast and frugal noncompensatory heuristic decision strategies. However, when the opportunity to compare the potential payoff is eliminated (Mondal & Traczyk, 2023a), people with higher numeracy fail to understand the difference in importance of the choice problems, resulting in impaired management of time and effort and less flexibility in strategy selection.

Furthermore, metacognitive factors (such as enhanced deliberation or higher confidence about the decisions made) also play an important part in aiding the decision process of highly numerate individuals to make better overall decisions. It has been demonstrated consistently that individuals with higher numeracy, in comparison to individuals with lower numeracy, are more confident when assessing the accuracy of their decisions (Ghazal et al., 2014) and spend significantly more time deliberating on choice problems (Ghazal et al., 2014; Mondal, 2021; Petrova et al., 2016; Traczyk, Sobkow, et al., 2018). Crucially, deliberation time is linked to the amount of information individuals with higher numeracy typically gather when facing a decision problem. In particular, studies employing the sampling paradigm have shown that individuals with higher numeracy tend to collect more samples per option and switch less frequently between options (Ashby, 2017; Traczyk, Lenda, et al., 2018), indicating a more effortful and comprehensive search strategy.

Taken together, these findings consistently demonstrate that individuals with higher statistical numeracy outperform those with lower numeracy in making choices, as they explore information more thoroughly and employ more deliberative processes to develop a better metacognitive understanding of choice problems. In the present study, we utilize the knowledge of these individual differences to examine the underlying cognitive mechanisms responsible for better choices under a resource-constrained task environment.

The Overview of the Present Study

In the present research, we tested whether individuals can comprehend the structure of the decision task and make recurring suboptimal choices that are more rewarding than a normatively superior strategy in the long run. Here, the normative decision strategy (i.e., the EV maximization model) serves as a benchmark for accurate behavior under risk that maximizes reward. Therefore, choices that do not follow the prediction from the normative model are considered suboptimal choices. We expect that, under time constraints, individuals can adapt their decision strategy to the task structure by making faster suboptimal (or random in terms of EV maximization) decisions that result in overall superior performance in the long run (e.g., earning more money). In addition, we explore whether individuals with higher statistical numeracy better understand the numeric and environmental complexities of task structures and are able to consistently make more suboptimal choices that not only approximate but also outperform the normative strategy. Through the analysis of their sampling behavior and verbal reports, our goal is to uncover the underlying metacognitive processes involved in making these decisions.

First, we validated our predictions through a simulation study. Second, we observed several key findings in three empirical studies using the sampling paradigm. In Study 1, we found that more EV-consistent choices and more effortful information processing (e.g., taking more samples, switching less between gambles) were predictors of higher total rewards. Study 2 revealed that making more random choices, in terms of the EV maximization principle, was associated with different sampling strategies. This was particularly evident in individuals with higher numeracy. Furthermore, participants who made more suboptimal choices under time constraints achieved a higher total reward. Study 3 replicated the effect of recurring irrational choices. Moreover, it showed that decision makers who earned a higher reward adaptively adjusted their decision strategy to the task's requirements and demonstrated a metacognitive understanding of the choice problem task.

Simulation Study

The simulation study demonstrates that a deliberate and normative decision strategy may not result in superior overall outcomes. Instead, we argue that making more suboptimal choices under the current task structure can lead to better overall performance (as measured by a higher total gain).

To support this claim, we generated responses (i.e., dichotomous choices between two gambles) from the same 10 choice problems used in the empirical studies (see Supplemental Table S5). The simulation was based on a sample of $N = 300$ subjects and resulted in 60,000 observations, varying in their EV consistency (i.e., the probability of selecting a gamble with a higher EV; 0, 0.25, 0.5, 0.75, 1) in making 40 choices (drawn from a set of 10 choice problems with replacement).

The simulation result is consistent with the prediction of recurring irrationality. As captured by Figure 2, we found that the "blue" decision maker would face 30 choice problems, given that they are following a more energy-intensive EV maximization strategy 75% of the time. However, the "orange" decision maker would earn more reward, compared to the "blue" decision maker, by facing 10 (i.e., 40 choice problems in total) problems more by randomly choosing between options (i.e., EV consistency of 50%). Existing literature indicates that there is an inverse relationship between the speed of decisions and the accuracy of those choices (Heitz, 2014; Liesefeld & Janczyk, 2019; Wickelgren, 1977). Therefore, assuming that making more EV-consistent choices requires more time, decision makers who make faster choices by deliberating less on the choice problem, sampling less information, and reducing their EV consistency will encounter more choice problems and would earn more rewards, under time constraints, than decision makers who make more accurate EV-consistent choices.

Further analysis of the simulation result indicates that the significant differences in mean reward (based on the choice problems used in this research) as a function of the EV consistency appeared after solving 30 choice problems. The algebraic solution applied to the current set of choice problems indicates that to receive the same expected reward, an "orange" decision maker, who chooses gambles randomly, should make five times more decisions in the same period of time compared to a "blue" decision maker who selects gambles with a higher EV in 75% of all choice problems.

In summary, making fast suboptimal choices can, in principle, give a superior overall reward than a slow and time-intensive normative strategy. In addition, by altering the task structure to encourage recurring suboptimal choices, we can assess whether skilled decision makers grasp the statistical structure of the task environment and are capable of shifting from their deliberate normative strategy to a faster suboptimal strategy to earn higher rewards. We validate these simulation results systematically in three empirical studies[2].

General Method

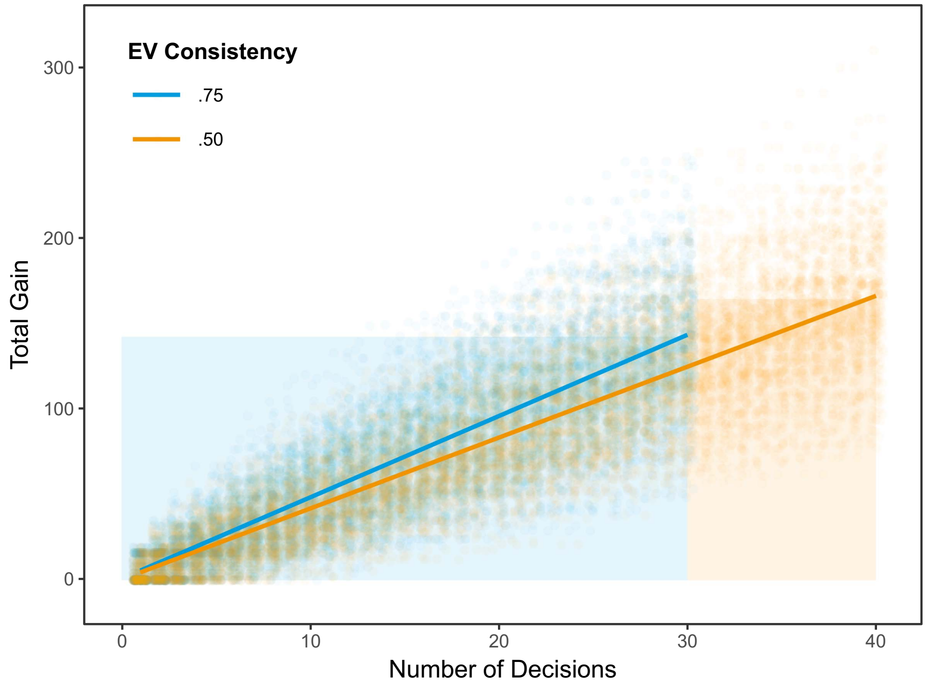
Participants

A diverse sample of N = 1,043 volunteers (51.2% females, M_{age} = 37.37, SD_{age} = 13.91), recruited via the Prolific platform, completed

[2] In addition, we conducted an auxiliary study, the details of which are reported in the Supplemental Materials.

Figure 2

Total Gain as a Function of the Number of Decisions Made With Different EV Consistency



Note. The blue line models decision makers who maximize EV in 75% of 30 decision problems, while the orange line models those who maximize EV in 50% of 40 decision problems (they make random choices). Dots illustrate simulated individual data. Blue and orange areas represent the maximum number of decisions (*x*-axis) and mean total gain (*y*-axis) for the two EV consistency conditions. EV = expected value. See the online article for the color version of this figure.

three online studies ($N_{\text{Study1}} = 350$, $N_{\text{Study2}} = 348$, $N_{\text{Study3}} = 345$). Participants were paid £1.88 for a study lasting approximately 15 min. In addition, they were informed that they would be compensated based on their performance on the decision task. Each participant received an additional £1 for every 200 points on top of the flat payment they would receive once they completed the task. Participants gave informed consent before the study. The study protocol was approved by the Ethics Committee at the SWPS University of Social Sciences and Humanities. Last, each participant can only participate in one of the three studies.

We applied the same exclusion criteria for all empirical studies[3]. Specifically, we analyzed data only from the participants who completed all the tasks. We also excluded repeated submissions from participants with the same identification number, those who did not make any choice to maximize EV, and those whose total gain on the whole task was 0 points. Based on these criteria, we excluded 10 participants in Study 1, 30 in Study 2, and 10 participants from Study 3. The majority of excluded participants accepted an invitation to the study but withdrew after starting the procedure, which was the main reason for exclusions.

Tasks

Examples of the experimental task, instructions, data sets, codes, and scripts of the present project can be found at https://osf.io/56xfa/ (Traczyk et al., 2024).

Decisions-From-Experience Task

In all studies, we used a decisions-from-experience task (DfE) to investigate information search and modulation in decision strategy (Hertwig & Erev, 2009; Hertwig et al., 2004;

[3] See preregistration at https://osf.io/k7tfx (Traczyk et al., 2022) and https://osf.io/u59fc (Mondal & Traczyk, 2023b).

Wulff et al., 2018). In the task, participants were presented with two boxes symbolizing binary two-outcome gambles with an unknown payoff distribution. Participants were informed that they could sample information from each gamble without limitations (see Supplemental Figure S1). For each selection of a gamble, an outcome drawn randomly from a given distribution was displayed for 400 ms. Participants were able to decide by themselves which distribution they wanted to sample from, when to switch between the gambles, and when to terminate exploration. When participants were ready to make a choice, they finished sampling and indicated which gamble they preferred by clicking the "Done" button below the boxes and then selected a gamble by clicking on the corresponding box. Feedback on their choice was provided immediately. In addition, information on their total current gain was presented in the screen's bottom-left corner (see Figure A1 for a snapshot of the task layout). We recorded participants' preferences, the number of decisions, the number of samples while exploring the gambles, time spent on exploration, and calculated the switching ratio (i.e., the ratio between the number of actual switches between gambles and the possible number of all switches given a total number of drawn samples).

Statistical Numeracy

We measured statistical numeracy using the Berlin Numeracy Test (BNT; Cokely et al., 2012). The BNT is a standardized psychometric instrument that efficiently measures objective numerical abilities, including statistical numeracy, risk literacy, and comprehension of probability. The BNT consists of mathematical tasks of varying difficulty, for example, "Imagine we are throwing a five-sided die 50 times. On average, out of these 50 throws, how many times would this five-sided die show an odd number (1, 3, or 5)?" In the three experiments, we used the computerized four-item BNT version. Items were presented sequentially in a fixed order. We took the sum of the correct answers on the BNT to calculate the measure of numeracy (from 0 to 4).

Procedure

The order of tasks (BNT and DfE) was counterbalanced across participants (order effects were not found). Prior to the DfE task, participants were familiarized with the mechanics of the DfE task and bonus payment mechanism using training trials. During the training trial, participants were first taught how to explore information hidden in the box and then asked to explore the two gambles to select their preferred gamble. Finally, participants were asked to answer correctly on two training trials to test their understanding of the general task. In particular, in the two consecutive choice problems, participants had to select a gamble that returned 0 points less frequently and the gamble that offered the highest possible profit, respectively. Tips concerning the mechanism of the DfE task were displayed on a computer screen during the training session. To proceed to the main DfE task, participants had to provide correct answers on the training task. They received feedback on the accuracy of their decisions and had to repeat the training task until they provided two correct responses (see Open Science Framework repository, Traczyk et al., 2024, for a video illustration).

The general procedure across all three studies was very similar. The critical difference concerned the time constraints and the different sets of choice problems used in the decision task (see Figure A1 for a summary of differences between studies). In Study 1, participants were explicitly informed that their task was to make 30 decisions in 30 choice problems without any time constraints. In Studies 2 and 3, participants were informed that the DfE task would last exactly 5 min. We additionally displayed a timer indicating the elapsed time during the DfE task (counting down from 5 min). Furthermore, in Study 3, participants were presented with choice problems from 120 unique binary choice problems consisting of two-outcome gambles in the gain domain and questions measuring fluid intelligence and metacognitive understanding of the choice task structure as well as employed decision strategies. In all studies, participants were informed that they would be additionally compensated based on their task performance. No other manipulated variables or blinding were involved in any of the studies.

Statistical Approach

To verify our hypotheses across the three studies, we first analyzed the relationships among measured variables using Pearson's coefficients, and we fitted multiple linear regression models predicting total gain (measured by points collected in the DfE task). The analysis was performed on aggregated data (i.e., participants as a unit of analysis). Our main predictors of interest were (a) the proportion of choices maximizing EV (i.e., EV consistency) and its interaction with numeracy measured by the BNT. We controlled measures of exploration behavior: the mean number of samples drawn, mean switching ratio, and mean exploration time. In each study, we fitted two models: one without control variables and another including them. All predictors were mean-centered.

Next, we employed Generalized Additive Modeling (GAM; Wood, 2006) to investigate the differences in exploration behavior between individuals with low and high numeracy as a function of consecutive choice problems that each participant solved. GAM is a modeling technique that captures complex nonlinear relationships between predictors/covariates and a response variable. In contrast to a simple linear regression model, GAM replaces β coefficients with flexible smooth functions constructed of multiple basis functions, allowing for nonlinear, wiggly relationships between variables. We used the long-data format (i.e., choice problems nested in participants as a unit of analysis) to fit nine separate GAMs for each study and response variable (i.e., number of samples, switching ratio, and exploration time). We fitted all GAMs with the restricted maximum likelihood method and selected thin plate regression splines as smoother. To account for intraindividual variability in responses, participants and choice problems were defined as random variables (Pedersen et al., 2019). The role of numeracy in each of the nine models was evaluated by comparing a null model (i.e., a response variable predicted by a smooth function of the number of solved choice problems; without numeracy) and an alternative model (i.e., a response variable predicted by a factor-smooth interaction of dichotomized numeracy score and the number of solved choice problems). We interpreted results with the Akaike information criterion (AIC) and visualizations. All analyses were performed using the mgcv (Wood, 2011) package in the R statistical environment (R Core Team, 2013).

Finally, we explored how participants' decision strategies evolved over time and identified the specific search strategies participants relied upon when making choices. To assess changes in strategy, we analyzed the proportion of EV-consistent choices throughout the study. In addition, we categorized participants based on the strategies that most accurately explained their choices: the EV maximization strategy, the PH (Brandstätter et al., 2006), and the maximax strategy (Coombs et al., 1970). In contrast to the EV maximization strategy, the PH posits that decision makers evaluate gambles by sequentially comparing their minimum gains, the probabilities of these minimum gains, and their maximum gains. The examination is terminated when the difference between the minimum gains exceeds 10% of the maximum gain or when the difference in probabilities of minimum gains is greater than 10% of the probability scale. Conversely, the maximax strategy disregards probability information, focusing solely on the maximum outcomes of gambles and selecting the one with the most attractive maximum outcome. The classification was based on the maximum likelihood approach (Pachur & Galesic, 2013; Rieskamp, 2008). We evaluated the goodness of fit of the three decision-making strategies using the G^2 measure:

$$G^2_{\text{Model}} = -2\sum_{i=1}^{N} \ln[f_i(y)]. \tag{1}$$

Where i refers to the choice problem, N to the total number of choice problems, and $f(y)$ to the probability the model predicts an individual choice y in a choice problem i. If Gamble A is selected, then $f(y_i) = p(A|B)$, and if Gamble B is selected then $f(y_i) = 1 - p(A|B)$. The lower the G^2 value, the better the strategy describes individual choices. Participants were classified as followers of a given strategy when G^2 for the strategy was lower than for other candidate strategies. If G^2 for a random strategy (i.e., when $f(y_i) = 0.5$) was the lowest, then the strategy was classified as random (or another unlisted strategy different from EV, PH, or maximax).

We also explored the dynamics of sampling by examining the changes in the sampling rate over time. The sampling rate is defined as the ratio between the number of samples taken and the time spent on sampling. Regarding the search strategies, we focused on the proportion of choices aligned with three distinct approaches: uncertainty-driven, value-driven, and discovery-driven strategies (Spektor & Wulff, 2023). For each participant and each choice problem where at least two samples were drawn, we normalized the number of samples to a range from 0 to 1. We then calculated the probability of selecting a gamble based on whether it had a higher experienced variance (uncertainty-driven strategy), a higher mean outcome (value-driven strategy), or fewer unique outcomes (discovery-driven strategy).

Study 1

The objective of Study 1 was to investigate the predictors of superior decision making in an unconstrained environment. To achieve this, participants were asked to solve exactly 30 choice problems using the DfE task. In this setup, there were no time constraints, allowing participants to explore each choice problem for as long as they needed.

Results and Discussion

Predictors of Overall Performance in the Decision Task

Tables representing descriptive statistics and Pearson's correlation coefficients for measures used in Study 1 are presented in the Supplemental Tables S3 and S4, respectively. In line with our preregistered hypothesis, total gain was related to the higher proportion of EV-consistent choices, $r(349) = .266, p < .001$. We also found that higher numeracy was related to higher EV consistency, $r(349) = .176, p < .001$, the number of samples drawn, $r(349) = .249, p < .001$, and exploration time, $r(349) = .145, p = .007$, but negatively associated with the switching ratio, $r(349) = -.262, p < .001$. We did not find a significant relationship between numeracy and total gain, $r(349) = .014, p = .395$.

To test whether numeracy moderated the effect of EV consistency on the total gain in the task, we fitted the participant-level multiple linear regression model (see Table 1). In the first step, we introduced only two predictors: BNT score and EV consistency. In the second step, we added control measures of exploration behavior (number of samples, switching ratio, and response time), and the interaction term between BNT score and EV consistency. In line with our

Table 1
Results of Linear Regression Models Predicting Total Gain in the Three Studies

Predictor	Total gain Estimates	[95% CI]	*p*	Total gain Estimates	[95% CI]	*p*
Study 1						
(Intercept)	135.76	[132.53, 138.99]	<.001	135.39	[132.11, 138.67]	<.001
EV consistency	69.94	[43.40, 96.48]	<.001	67.59	[39.70, 95.48]	<.001
Numeracy	−0.87	[−3.53, 1.79]	.522	−1.14	[−3.95, 1.66]	.424
Number of samples				−0.12	[−0.82, 0.59]	.747
Switching ratio				4.71	[−9.74, 19.16]	.522
Response time				0.56	[−0.10, 1.22]	.098
EV Consistency × Numeracy				13.68	[−8.17, 35.53]	.219
		R^2/R^2 adjusted = .072/.067			R^2/R^2 adjusted = .087/.071	
Study 2						
(Intercept)	174.73	[164.22, 185.25]	<.001	175.61	[167.37, 183.86]	<.001
EV consistency	−291.52	[−378.76, −204.28]	<.001	−124.49	[−200.39, −48.60]	.001
Numeracy	8.57	[0.13, 17.00]	.046	9.43	[2.70, 16.16]	.006
Number of samples				−3.70	[−5.61, −1.78]	<.001
Switching ratio				59.32	[23.85, 94.78]	.001
Response time				−3.20	[−5.14, −1.25]	.001
EV Consistency × Numeracy				−61.48	[−117.67, −5.29]	.032
		R^2/R^2 adjusted = .119/.114			R^2/R^2 adjusted = .466/.456	
Study 3						
(Intercept)	304.13	[282.16, 326.10]	<.001	304.34	[288.52, 320.17]	<.001
EV consistency	−317.00	[−533.73, −100.27]	.004	25.75	[−135.80, 187.30]	.754
Numeracy	15.13	[−2.19, 32.44]	.087	13.07	[0.35, 25.79]	.044
Number of samples				−11.15	[−15.44, −6.86]	<.001
Switching ratio				20.77	[−49.54, 91.09]	.562
Response time				−11.24	[−16.33, −6.15]	<.001
EV Consistency × Numeracy				22.86	[−102.57, 148.30]	.720
		R^2/R^2 adjusted = .034/.028			R^2/R^2 adjusted = .507/.499	

Note. CI = confidence interval; EV = expected value.

preregistered hypothesis in Study 1, we found that when participants were presented with a fixed number of choice problems, better performance on the decision task (measured by the total gain) was related to their EV consistency, $b = 67.58$, $t(343) = 4.76$, $p < .001$. There was no effect of numeracy, $b = -1.14$, $t(343) = -0.80$, $p = .424$. We did not find any other statistically significant predictors of total gain.

Investigating Adaptive Exploration

We further explored how people with low and high numeracy processed information about choice problems. Mainly, we were interested in such exploration measures as the number of samples drawn, switching frequency (i.e., how frequently participants switched between two probability distributions), and response time (i.e., time spent exploring outcomes of a particular choice problem). A summary of nine GAMs predicting these three outcome measures across the three studies is presented in Table 2. We also visualized these relationships in Figure 3 by plotting fitted models upon raw data. In Study 1, we observed results similar to those of previous studies (Ashby, 2017; Traczyk, Lenda, et al., 2018). That is, people with higher numeracy, in comparison to people with lower numeracy, drew more samples (Figure 3, Panel A.1.), switched less frequently between options (Figure 3, Panel B.1.), and spent more time on choice problem exploration (Figure 3, Panel C.1.). Despite the fact that participants, in general, sampled less, switched more frequently, and spent less time on exploration as a function of solving consecutive choice problems (a fixed number of 30 choice problems), there were no clear differences in these tendencies between participants with higher and lower numeracy. The findings concerning sample size and response time were corroborated by analyzing the change in AIC between models without and with numeracy that was negligible (ΔAIC is -2.81 and -4.97, respectively). In the case of the switching ratio, more numerate participants expressed more consistent (i.e., more linear) change in their switching policy as a function of the consecutive choice problems (ΔAIC $= 25.62$, indicating a better fit of a model including interaction with numeracy).

We found that most participants were classified as following the EV maximization strategy (61%, 213 participants), while 7% (23 participants) and 32% (114 participants) were classified as following the PH and random strategy, respectively. The differences were statistically significant, $\chi^2(2) = 154.81$, $p < .001$. The followers of the maximax strategy were not identified. Next, to further understand how individuals make choices and explore choice problems, we examined the changes in the likelihood of selecting a gamble with a higher EV (i.e., the proportion of choices maximizing EV across the consecutive choice problems solved). To assess whether choices followed the EV maximization or random strategy, we calculated 95% Clopper-Pearson CI (confidence interval) derived from a binomial distribution for consecutive binary choice problems. When point estimates representing a proportion of choices maximizing EV are within the 95% CI, we can conclude that the decision strategy is statistically indistinguishable from a random selection strategy (see Supplemental Figure S2 for the proportion of choices consistent with the predictions of the PH). Moreover, we calculated the sampling ratio—the total number of samples in a given choice problem divided by the time spent exploring. For the conciseness of the latter analysis, we averaged the dependent variables across five bins for each choice problem and participant. Higher values of this measure suggest more dynamic sampling (e.g., a sampling ratio value of 1 signifies that one sample is drawn every second).

Our primary interest was in discerning whether individuals with varying levels of numeracy altered their choice strategies over time. To test the significance of this effect, we used linear mixed models with numeracy and consecutive choice problems as predictors, including their interaction term, and participants as a random-intercept effect, with a specific focus on the interaction term. In Study 1, as captured by Figure 4A, the interaction effect was statistically significant, $b = 0.001$, $t(10{,}150) = 4.033$, $p < .001$. Choices made by participants with higher numeracy followed the EV maximization strategy (the point estimate was outside 95% CI after 30 choices), while participants with lower numeracy made fewer EV-consistent choices and followed a random strategy. As captured in Figure 5A, the effect was also significant for the sampling ratio, $b = 0.007$, $t(1{,}398) = 2.717$, $p = .007$. Individuals with higher numeracy tended to sample information more dynamically and consistently throughout the task, whereas those

Table 2

The Summary of the Hierarchical Generalized Additive Models Predicting the Log of the Number of Samples, Switching Ratio, and the Log of Response Time With a Participant- and Choice Problem-Level Random Intercept Effects

	Number of sample (log)				Switching ratio				Response time (log)			
Parametric coefficient	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*
Intercept	2.14	0.07	32.96	<.001	0.37	0.02	19.83	<.001	2.16	0.05	44.15	<.001
Numeracy	0.46	0.08	6.00	<.001	−0.15	0.03	−5.42	<.001	0.27	0.06	4.23	<.001
Smooth term and random effect	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*
Smooth terms												
Numeracy (low)	5.16	6.28	83.54	<.001	5.64	6.79	21.28	<.001	7.33	8.74	106.79	<.001
Numeracy (high)	4.04	4.99	75.50	<.001	1	1	174.18	<.001	5.74	7.02	97.22	<.001
Random effects												
Participants	339.36	348	39.30	<.001	339.60	348	47.24	<.001	339.35	348	39.22	<.001
Choice problem	8.83	9	52.91	<.001	8.33	9	13.02	<.001	8.75	9	34.92	<.001
Study 1			Deviance explained (%)	.63			Deviance explained (%)	.63			Deviance explained (%)	.62
			AIC	19784.43			AIC	−4359.95			AIC	15532.34
			ΔAIC	−2.81			ΔAIC	25.62			ΔAIC	−4.97

	Number of sample (log)				Switching ratio				Response time (log)			
Parametric coefficient	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*
Intercept	1.72	0.06	31.25	<.001	0.39	0.02	21.56	<.001	1.78	0.04	45.3	<.001
Numeracy	−0.01	0.07	−0.19	.846	−0.06	0.03	−2.33	.020	−0.06	0.05	−1.11	.268
Smooth term and random effect	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*
Smooth terms												
Numeracy (low)	6.78	7.74	284.55	<.001	4.67	5.44	37.56	<.001	6.84	7.39	469.05	<.001
Numeracy (high)	7.80	8.95	306.33	<.001	4.15	5.06	54.14	<.001	8.01	8.62	433.5	<.001
Random effects												
Participants	336.82	346	39.34	<.001	329.86	345	27.32	<.001	336.71	346	31.96	<.001
Choice problem	8.70	9	30.54	<.001	6.94	9	3.69	<.001	8.57	9	21.01	<.001

(*table continues*)

Table 2 (*continued*)

Smooth term and random effect	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*
Study 2			Deviance explained (%)	.72			Deviance explained (%)	.55			Deviance explained (%)	.74
			AIC	25377.74			AIC	33.57			AIC	17021.52
			ΔAIC	54.36			ΔAIC	46.65			ΔAIC	24.01

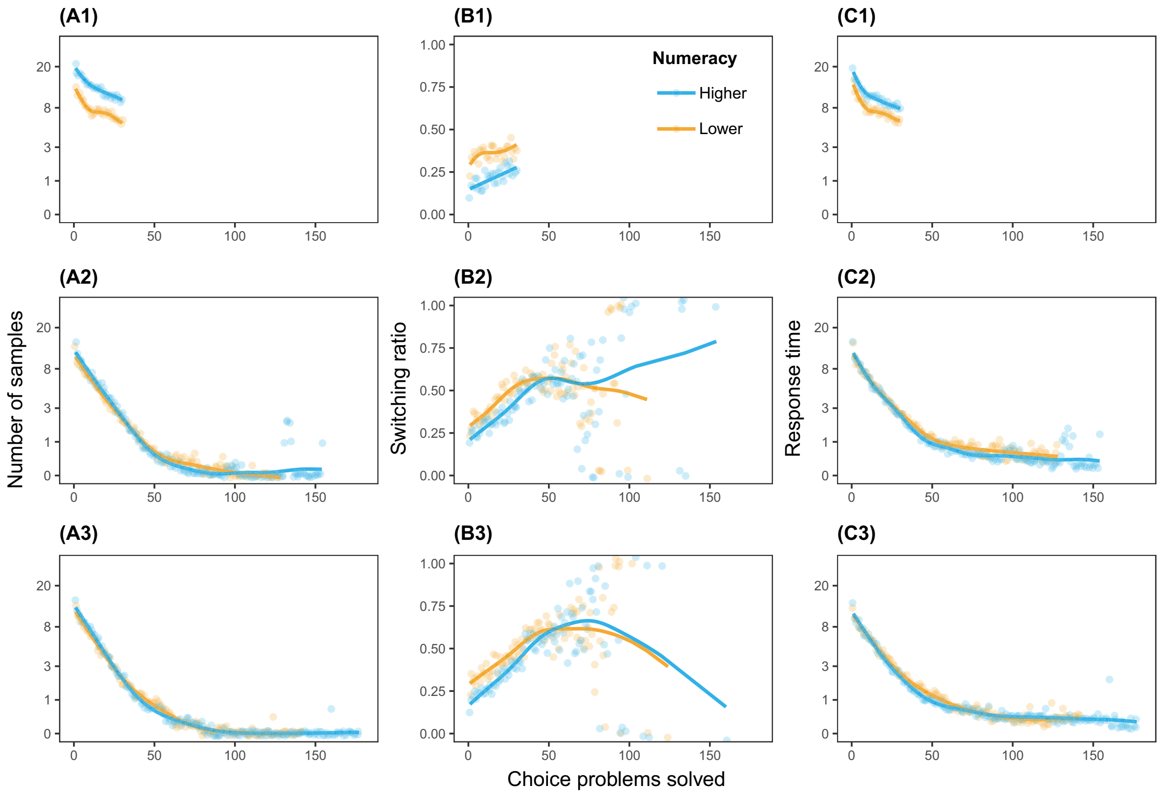
	Number of sample (log)				Switching ratio				Response time (log)			
Parametric coefficient	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*	Estimate	*SE*	*t*	*p*
Intercept	1.69	0.05	35.3	<.001	0.39	0.02	22.41	<.001	1.73	0.04	45.38	<.001
Numeracy	0.01	0.07	0.17	.862	−0.08	0.03	−3.20	.002	−0.05	0.06	−0.88	.378

Smooth term and random effect	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*	*edf*	*df*	*F*	*p*
Smooth terms												
Numeracy (low)	6.64	7.55	332.88	<.001	6.45	7.15	35.11	<.001	6.49	7.02	471.32	<.001
Numeracy (high)	7.83	9.05	313.36	<.001	4.28	5.24	68.60	<.001	7.68	8.44	437.43	<.001
Random effects												
Participants	333.43	343	38.83	<.001	328.2	343	29.01	<.001	335.04	343	39.92	<.001
Choice problem	74.89	119	2.27	<.001	9.29	119	0.09	.237	69.6	119	1.94	<.001
Study 3			Deviance explained (%)	.73			Deviance explained (%)	.58			Deviance explained (%)	.76
			AIC	27739.4			AIC	−1124.83			AIC	18187.52
			ΔAIC	48.58			ΔAIC	30.42			ΔAIC	58.94

Note. Model fit was evaluated with explained deviance (%) and Akaike information criterion (AIC). Improvement in model fit was evaluated by the ΔAIC (difference between AICs for models with and without numeracy). *SE* = standard error; *edf* = effective degrees of freedom.

Figure 3

Differences in Adaptive Exploration Between Participants With Lower and Higher Numeracy



Note. Predicted log number of samples (Panel A), switching ratio (Panel B), and log response time (Panel C) as a function of dichotomized numeracy level (blue lines—higher numeracy, orange lines—lower numeracy) and the number of choice problems solved (on the x-axis) in Studies 1, 2, and 3 (plotted in following rows). Models were fitted with generalized additive modeling (GAM). Dots represent the means of raw data. The scale for the number of samples and response time was converted back from the log to the original scale. See the online article for the color version of this figure.

with lower numeracy exhibited less dynamic sampling, which even decreased over time.

To delve deeper into search strategies, we conducted an analysis of the proportion of samples allocated to the option with the highest variance (variance-driven search), the highest experienced mean (value-driven search), or the greatest number of unobserved outcomes (discovery-driven search). This analysis was conducted in relation to the relative position within the sampling sequence. To achieve this, we transformed the samples in each choice problem to reflect the relative position of a given sample within the entire choice problem, irrespective of the total number of samples drawn (Spektor & Wulff, 2023). For each choice problem and participant, we divided the sequence into ten equal bins for detailed analysis.

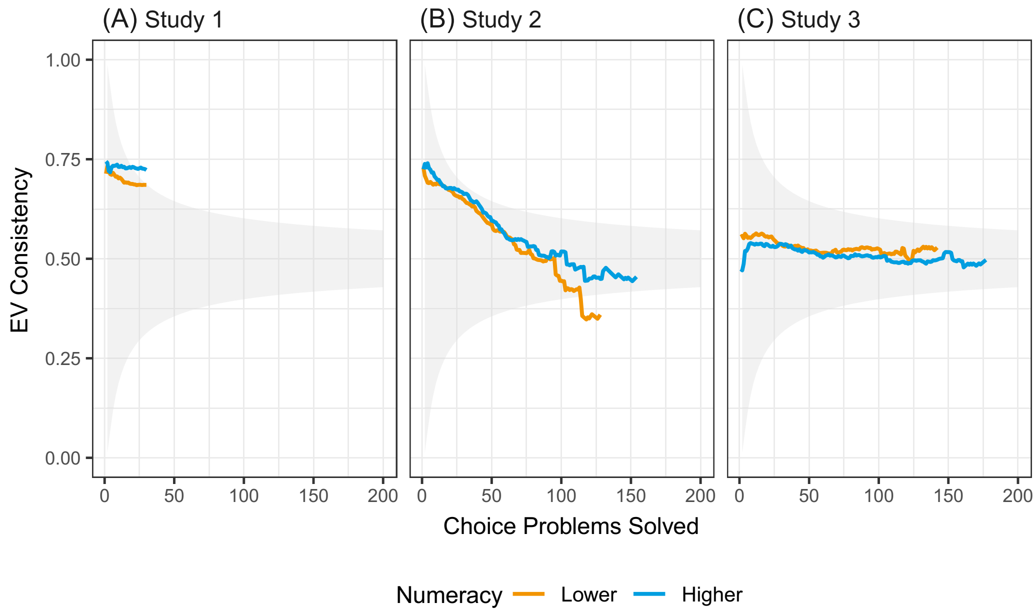
Figure 6A illustrates that participants in Study 1 showed a preference for sampling from gambles perceived to have unobserved outcomes. This inclination became more pronounced midway through the search process. Notably, individuals with higher numeracy were more inclined towards a discovery-driven search strategy. The other two strategies, value-driven and variance-driven, did not play as significant a role in guiding the search. Participants showed a slight tendency to allocate more samples to options with lower variance and value.

Summary

In summary, Study 1 revealed that in situations without environmental constraints, higher total rewards were associated with choices that maximized EV. Enhanced task performance correlated with a more thorough search process, characterized by intensive sampling, fewer switches between gambles, and increased time

Figure 4

The EV Maximization Strategy (EV Consistency) as a Function of the Number of Choice Problems Solved and Numeracy



Note. The shaded regions denote the 95% CI derived from a binomial distribution for consecutive binary choice problems. The point estimates (depicted as lines) within the 95% CI suggest that the decision strategy is statistically indistinguishable from a random selection strategy. CI = confidence interval; EV = expected value. See the online article for the color version of this figure.

spent exploring. Notably, while participants with higher numeracy did not necessarily achieve higher rewards, they were more likely to make choices that maximized EV and engaged in more rigorous exploration of the choice problems. Interestingly, individuals with higher numeracy, in contrast to their less numerate counterparts, exhibited more dynamic and consistent sampling across all choice problems. They also showed a greater tendency to allocate samples to gambles where they believed not all outcomes had been observed as described by the discovery-driven search. In our subsequent study, we aimed to explore how imposing time constraints on the choice environment would affect decision making and exploration strategies. Specifically, we investigated whether these time limits would lead to changes in strategy that result in improved overall performance, potentially due to the adoption of recurring suboptimal choices.

Study 2

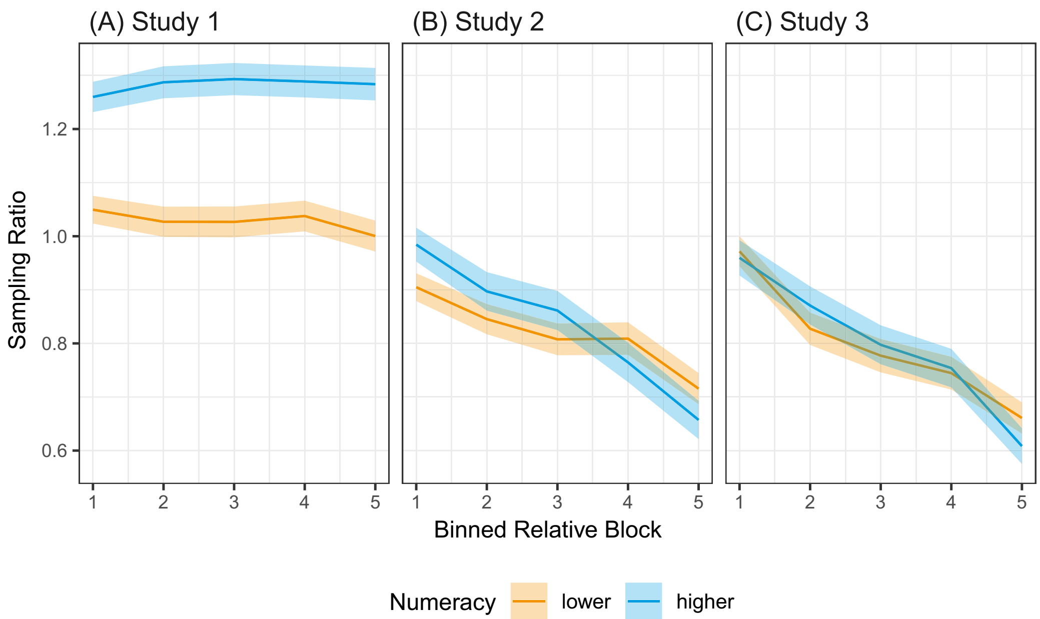
In Study 2, we introduced a time-constrained framework. Unlike in Study 1, participants were not aware of the exact number of choice problems they would encounter. Instead, they were informed that the entire task would span precisely 5 min. A timer, displayed at the bottom-right corner of the screen, indicated the passing time throughout the decision-making task. Our objective in this task was not to induce subjective feelings of stress typically associated with time pressure. Instead, we aimed to establish external time limits within which the decision task needed to be completed.

We hypothesized that skilled decision makers, who in an unconstrained environment employ more thorough search strategies to make choices that maximize EV, would adapt to this time-limited task and shift to quicker and more random strategies in terms of EV maximization. This approach, although seemingly suboptimal on a per-choice basis, was anticipated to lead to recurring suboptimal choices that, cumulatively, would result in superior overall performance.

Recent behavioral and neuroscience findings seem to support this general prediction, suggesting that decision makers, under constraints, balance the costs associated with increasing accuracy of

Figure 5

The Sampling Ratio (the Number of Samples Divided by the Sampling Time) Across the Three Studies Plotted Against the Choice Problem's Position in the Decision Task (Binned Relative Block)



Note. Orange and blue lines represent participants with lower and higher numeracy, respectively. The shaded areas around each line denote the 95% confidence intervals. See the online article for the color version of this figure.

their decisions against the diminishing benefits gained as a result of it—they are resource rational or optimally efficient (Glimcher, 2022; Ho et al., 2022; Mastrogiuseppe & Moreno-Bote, 2022). For instance, people adapt their behavior in a complex problem-solving task and use heuristic planning strategies to behave in an optimal manner when resources are constrained (Callaway et al., 2022). A reanalysis of eye-tracking data has revealed that attentional bias to a low-EV option was related to a higher reward rate—the reward that could be obtained per unit of time invested in a choice problem (Zilker, 2022). Furthermore, it was demonstrated that high-value decisions were being made faster and with greater accuracy than low-value decisions (Shevlin et al., 2022), which undermines the predictions of normative decision models.

Results and Discussion

Predictors of Overall Performance in the Decision Task

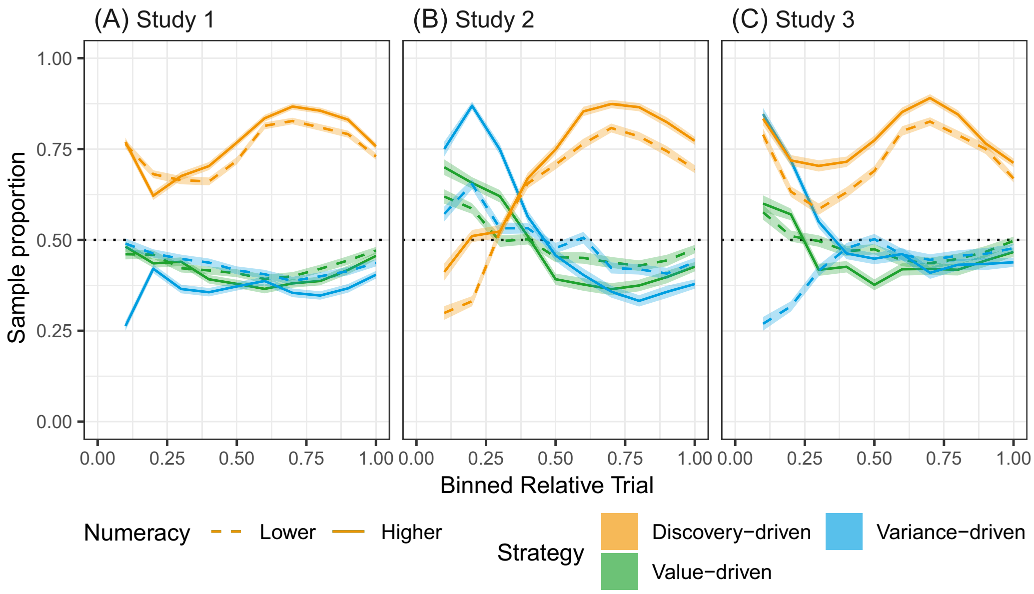
Tables with descriptive statistics and Pearson's correlation coefficients for measures used in Study 2 are presented in the Supplemental Tables S6 and S7, respectively. In contrast to Study 1, total gain was related to the lower proportion of EV-consistent choices, $r(347) = -.330$, $p < .001$, the lower number of samples drawn, $r(347) = -.630$, $p < .001$, and exploration time, $r(347) = -.611$, $p < .001$, but positively associated with the switching ratio, $r(347) = .358$, $p < .001$.

Higher numeracy was weakly related to higher total gain, $r(347) = .096$, $p = .037$, but not to higher EV consistency, $r(347) = .016$, $p = .382$. Participants with higher numeracy also made more decisions, $r(347) = .092$, $p = .044$, and the number of decisions was the strongest predictor of overall reward in the task, $r(347) = .939$, $p = .382$.

The regression analysis (Table 1) revealed that when making choices within a fixed period of time, participants with higher numeracy performed better in the task, $b = 9.43$, $t(340) = 2.76$, $p = .006$, and the effect remained significant even when the model was adjusted by exploration measures. Decision makers who collected more points in the task also sampled less information, $b = -3.70$, $t(340) = -3.79$, $p < .001$, switched between gambles more frequently, $b = 59.32$, $t(340) = 3.29$, $p = .001$, and spent less time on the

Figure 6

The Distribution of Samples Among the Three Different Search Strategies: Value-Driven (Green), Variance-Driven (Blue), and Discovery-Driven (Orange), for Each Option Across Three Studies



Note. This distribution is based on the sample's position in the sequence of trials, which has been organized into bins for analysis. Dashed and solid lines represent participants with lower and higher numeracy, respectively. The shaded areas around each line denote the 95% confidence intervals. See the online article for the color version of this figure.

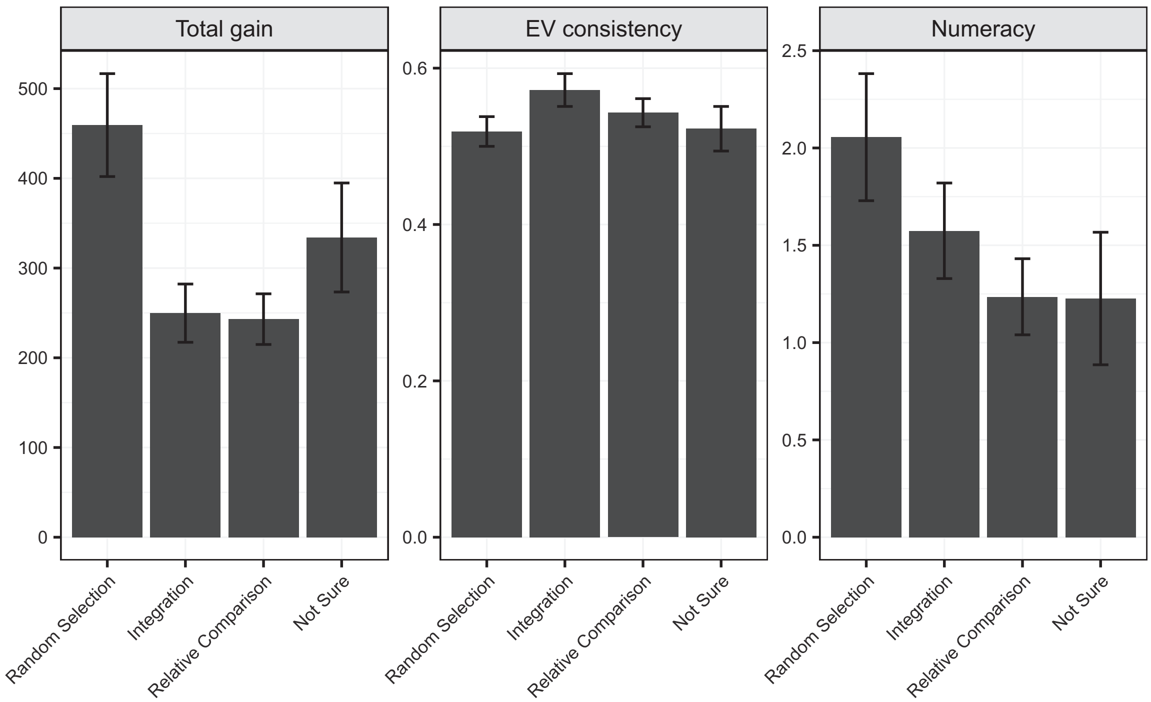
exploration, $b = -3.20$, $t(340) = -3.23$, $p = .001$. In contrast to Study 1, EV consistency was negatively related to total gain, $b = -124.49$, $t(340) = -3.22$, $p = .001$. We found a significant interaction between EV consistency and numeracy, $b = -61.48$, $t(340) = -2.15$, $p = .032$, indicating that the slope for the relationship between EV consistency and total gain is moderated by numeracy. Further examination of the interaction effect indicated that the greatest predicted gain was anticipated among individuals with higher numeracy who were inclined to make random choices.

Investigating Adaptive Exploration

To probe this interaction further, we fitted GAMs to investigate whether there are specific patterns of sampling behavior between individuals with higher and lower numeracy that can explain why participants made more choices in a fixed period. Participants with higher numeracy changed their strategy significantly in comparison to Study 1. They explored choice problems more dynamically. That is, they drew fewer samples, spent less time on exploration, and switched between options more frequently. As a consequence of such behavior, they saved a portion of time at the end of the task to solve more choice problems and earn more points (see the length of the orange and blue lines in Figure 3, Panels A.2., B.2., and C.2). For all models fitted to data collected in Study 2, we found a substantial increase in ΔAIC of 54.36, 46.65, and 24.01, suggesting that introducing different smooths for high and low-numeracy groups increased model fit.

In comparison to Study 1, the number of participants classified as followers of the random strategy was higher (41%, 141 participants), and there were fewer participants classified as following the EV strategy (48%, 170 participants). This change (between Studies 1 and 2) was statistically significant, $\chi^2(1) = 7.290$, $p = .007$. There were 37 participants (11%) classified as following the PH strategy and no participants who followed the maximax strategy. Differences between the proportions of participants using different strategies were again significant, $\chi^2(2) = 84.33$, $p < .001$. In our next analysis, we examined whether participants altered their EV maximization strategy across successive choice problems. We observed (Figure 4B) that the likelihood that

Figure 7
Mean Total Gain, EV Consistency, and Numeracy Score as a Function of the Reported Strategy Classification in Study 3



Note. Error bars represent the 95% confidence intervals. EV = expected value.

participants maximized EV was higher at the task's onset, but this tendency diminished over time, $b = -0.001$, $t(13{,}950) = -22.803$, $p < .001$. Numeracy did not significantly influence this trend, $b = 0.001$, $t(13{,}950) = 1.505$, $p = .132$. Participants with higher numeracy made random choices throughout the study (as denoted by the inspection of the 95% CI), while participants with lower numeracy made more choices that were not predicted by the EV maximization strategy.

Further analysis of sampling dynamics revealed a significant effect related to the number of choice problems (Figure 5B). Initially, participants tended to sample more dynamically, but the sampling rate decreased with each subsequent choice problem, $b = -0.021$, $t(1{,}398) = -3.266$, $p = .001$. Intriguingly, numeracy influenced this pattern, $b = -0.011$, $t(1{,}398) = -3.357$, $p < .001$. Individuals with higher numeracy started by sampling more dynamically than those with lower numeracy, but then shifted their strategy to sample less frequently.

A more detailed examination of search strategies (Figure 6B) indicated that individuals with higher numeracy initially focused on uncertainty exploration (variance-driven search). As the task progressed, they adaptively switched to a strategy aimed at discovering more unobserved outcomes (discovery-driven search). Their tendency to adopt these strategies was more pronounced compared to individuals with lower numeracy.

Summary

Study 2, conducted in a time-constrained environment where participants had limited time to make as many choices as possible, revealed interesting insights. Participants who made more random choices without strictly following the EV maximization policy were able to save time and make more decisions, ultimately leading to higher overall rewards. This suggests that recurring suboptimal choices can paradoxically result in superior decision-making outcomes. While individuals with higher numeracy did achieve slightly higher rewards on average, the effect was not strong. However, more skilled decision makers exhibited adaptability in modifying their search strategies. Notably, they tended to make more random choices as the task progressed. In contrast to their less numerate counterparts, they flexibly adjusted their sampling dynamics over time and strategically employed different search strategies to better understand the task's structure and earn higher rewards.

Study 3

In Study 2, we limited the experiment to only 10 choice problems. As a result, participants who made more choices had the opportunity to solve the same problems multiple times, potentially basing their decisions on memory. Furthermore, while we noted various search strategies used by participants to achieve higher rewards, it remains unclear if those who performed better in the task developed a metacognitive understanding of the structure of the choice problems and the environment. Addressing these limitations was the primary goal of Study 3.

Materials and Procedure

Choice Problems

We used a new set of 120 unique binary choice problems consisting of two-outcome gambles in the gain domain. The design of these choice problems aimed to more sharply differentiate between a compensatory strategy, which involves weighting and summing operations as captured by expectation models like EV or CPT (Tversky & Kahneman, 1992), and a heuristic strategy, as represented by the PH model (Brandstätter et al., 2006). In the current set of choice problems, the two decision-making models predicted opposite choices in 91% of instances, with CPT predictions always aligning with the EV maximization policy. This approach was inspired by previous studies (Mondal, 2021; Pachur & Spaar, 2015; Traczyk, Sobkow, et al., 2018).

Materials

To gauge participants' metacognitive understanding of the structure of the choice problems and the decision-making environment, we first asked them to provide subjective ratings of their performance in the task ("How would you rate your performance in the choice task?") by using a slider scale ranging from −50 (*very bad*) to 50 (*very good*). Following this, an open-ended question was posed, prompting participants to elaborate on the strategies they employed to earn higher rewards ("Please describe what strategies you used to earn a higher reward in the decision task. How did you make your decisions?"). We assumed that participants' metacognitive understanding of the structure of the choice problems and the decision-making environment is reflected in their accurate judgment of their own decision strategies.

To assess fluid intelligence, we utilized four matrix reasoning items from the International Cognitive Ability Resource measure (ICAR; Condon & Revelle, 2014). These items were akin to the ones used in Raven's Advanced Progressive Matrices (Raven, 2000). Each reasoning problem was presented as a three-by-three matrix of elements with one element missing. Participants were tasked with identifying the underlying rule of the matrix and selecting one of six response elements that conformed to this rule. The possible scores ranged from 0 to 4 points, with higher scores indicating greater fluid intelligence.

Procedure

The procedure for the DfE task in this study was consistent with that of Study 2. Unlike Studies 1 and 2, the choice problems in Study 3 were modified to address previously identified limitations. However, the ratio of bonus payments remained consistent across all three studies. The order of tasks (i.e., DfE task, BNT, ICAR) was counterbalanced. Questions regarding participants' metacognitive understanding were posed after the DfE task but before informing participants of their total gain. After completing the BNT, participants were additionally queried about their familiarity with the items presented, specifically asking if they had previously encountered the BNT.

Results and Discussion

Predictors of Overall Performance in the Decision Task

Tables representing descriptive statistics and Pearson's correlation coefficients for measures used in Study 3 are presented in the Supplemental Tables S9 and S10, respectively. Similarly to

Study 2 and in contrast to Study 1, total gain was related to the lower proportion of EV-consistent choices, $r(344) = -.160$, $p = .003$, the lower number of samples drawn, $r(344) = -.688$, $p < .001$, and exploration time, $r(344) = -.676$, $p < .001$, but positively associated with the switching ratio, $r(344) = .305$, $p < .001$.

As we predicted in our preregistered hypothesis, we found a weak but statistically significant positive relationship between numeracy and total gain, $r(344) = .103$, $p = .028$, but not between numeracy and higher EV consistency, $r(344) = -.073$, $p = .089$. Participants with higher numeracy also made more decisions, $r(344) = .101$, $p = .030$, and the number of decisions was the strongest predictor of overall reward in the task, $r(344) = .963$, $p < .001$. There was no statistically significant relationship between numeracy and subjective ratings of performance, $r(344) = .046$, $p = .394$. Interestingly, we found that this measure of metacognitive understanding was significantly correlated with total gain, $r(344) = .464$, $p < .001$, and the number of decisions made, $r(344) = .412$, $p < .001$. Participants who rated their own performance as better after completing the task indeed achieved higher total rewards.

While there was a significant correlation between numeracy and fluid intelligence, as measured by the ICAR test, $r(344) = .307$, $p < .001$, fluid intelligence did not predict total gain, EV consistency, nor the number of decisions made ($p = .590$, $p = .820$, $p = .284$, respectively).

Similarly to Study 2, the regression analysis indicated that within a fixed time frame, lower EV consistency initially predicted a higher total gain, $b = -317.00$, $t(342) = -2.877$, $p = .004$. However, as captured by Table 1, this effect became nonsignificant when additional control measures were incorporated into the model. Participants with higher numeracy performed better in the task, $b = 13.07$, $t(338) = 2.022$, $p = .044$, after adjusting the model for exploration measures. In addition, we discovered that individuals who accumulated more points sampled less information, $b = -11.15$, $t(338) = -5.107$, $p < .001$, and spent less time exploring, $b = -11.24$, $t(338) = -4.341$, $p < .001$. Contrary to the findings in Study 2, there was no significant interaction between EV consistency and numeracy in this study, $b = 22.86$, $t(338) = 0.359$, $p = .720$.

Crucially, when we adjusted the models to account for the ICAR scores, we observed that the abovementioned effects not only persisted but became more pronounced (see Supplementals Tables S11 and S12). Moreover, ICAR scores did not predict total gain, $b = -9.06$, $t(337) = -1.315$, $p = .190$. In addition, familiarity with the BNT items (84% of participants declared that they encountered the BNT for the first time) did not change the observed pattern of results we reported.

Investigating Adaptive Exploration

In our subsequent investigation, we used GAM analysis to understand how participants processed choice problems. The patterns of sampling and time management revealed by participants (as shown in Figure 3, Panel A3 for sampling and Panel C3 for response time) were similar to those observed in Study 2. This suggests that individuals with higher numeracy explored both faster and more dynamically, enabling them to make more decisions. Specifically, compared to those with lower numeracy, these individuals drew fewer samples and spent less time exploring. However, unlike in Study 2, their switching behavior between options was more akin to that of participants with lower numeracy. As a result of such strategic behavior, they managed to save time towards the end of the task, which allowed them to solve additional choice problems and accumulate more points. Furthermore, for all models fitted to the data from Study 3, we observed a significant increase in ΔAIC of 48.58, 30.42, and 58.94 (see Table 2). This increase indicates that adding different smooths for high- and low-numeracy groups substantially improved the fit of the models.

The number of participants classified as followers of the random strategy was higher (69%, 238 participants), and there were fewer participants classified as following the EV strategy (22%, 76 participants). This change (between Studies 1 and 3) was statistically significant, $\chi^2(1) = 108.407$, $p < .001$. There were 31 participants (9%) classified as following the PH strategy and no participants who followed the maximax strategy. Differences between the proportions of participants using different strategies were again significant, $\chi^2(2) = 206.14$, $p < .001$. In our subsequent analysis, we aimed to determine if participants altered their choice strategy. Our analysis revealed that participants made fewer EV-consistent choices as the choice task progressed, $b = -0.001$, $t(14{,}980) = -8.477$, $p < .001$. (Figure 4C). In addition, more numerate participants were less likely to make more EV-consistent choices, $b = -0.037$, $t(349.4) = -2.465$, $p = .014$, and numeracy moderated the effect of the number of choice problems solved on EV consistency, $b = 0.001$, $t(14{,}980) = 12.285$, $p < .001$.

Crucially, the analysis of the sampling rate replicated the findings from Study 2 (Figure 5C), demonstrating that the tendency to sample more dynamically decreased over time, $b = -0.01$, $t(1{,}398) = -6.039$, $p < .001$. Once again, numeracy was found to influence this pattern, $b = -0.012$, $t(1{,}398) = -3.341$, $p < .001$. Participants with higher numeracy initially sampled more dynamically compared to those with lower numeracy but then adapted their strategy to sample less frequently over time.

We also successfully replicated the key findings from Study 2 concerning the use of search strategies (Figure 6C). Notably, participants with higher numeracy tended to rely more on discovery-driven search strategies compared to those with lower numeracy, especially as sampling progressed. Similar to the patterns observed in Study 2, individuals with higher numeracy initially focused on exploring uncertainty through a variance-driven search strategy and then adaptively changed their strategy.

Metacognitive Understanding

Finally, we directly investigated whether participants developed a metacognitive understanding of the decision environment and the structure of choice problems and how this understanding related to their overall reward and numeracy. To assess this, we analyzed open-ended questions about the strategies participants reported using to solve decision problems. The analysis of qualitative data was preregistered (Mondal & Traczyk, 2023b). This analysis revealed four distinct categories: (a) Random Selection (this category was used when participants indicated they did not follow any specific strategy but instead made decisions rapidly and randomly, clicking as fast as possible), (b) Integration (responses were classified as "Integration" when participants described an approach that combined both outcomes and probabilities to identify the most suitable or attractive option. This method aligns with choices consistent with EV maximization), (c) Relative Comparison (in this category, participants focused on either higher frequency or higher outcomes at one time; responses were classified as "Relative" when the strategy involved concentrating solely on either outcome or probability, differentiating it from "Integration," where both factors are considered), and (d) Not Sure (this classification was for responses that did not informatively fit into any of the above three categories. "Not Sure" represented an ambiguous or undefined strategy).

We engaged four independent judges to categorize each participant's response into one of the four predefined categories. The level of agreement among the judges, as measured by Fleiss' κ, was .509, 95% CI [.484, .534], $p < .001$. This indicates a moderate but statistically significant level of agreement among the judges. The final classification of responses was determined based on the majority opinion of these judges. In instances where there was an equal split among the categories, an additional expert made the final decision.

A one-way analysis of variance, with strategy classification as the independent variable and total gain as the dependent variable, demonstrated that metacognitive strategies significantly predicted the total reward in the task, $F(3, 341) = 22.581$, $p < .001$, $\eta^2 = 0.166$ (Figure 7). Post hoc tests, adjusted with the Holm correction, revealed that participants who adopted the Random Selection strategy achieved significantly higher rewards compared to those who chose other strategies (all p values < .001). In addition, when considering EV consistency, we observed a statistically significant main effect, $F(3, 341) = 4.867$, $p = .003$, $\eta^2 = 0.041$. Participants employing the Integration strategy maximized EV more frequently than those using the Random Selection strategy ($p = .004$) and the unidentified (Not Sure) strategies ($p = .020$), but no significant differences were found compared to the Relative Comparison strategy ($p = .138$). Moreover, the selection of strategy was related to numeracy, $F(3, 341) = 7.621$, $p < .001$, $\eta^2 = 0.063$. The mean BNT score was significantly higher among participants who made random choices compared to all other strategies (all post hoc p values < .05). Regarding fluid intelligence measured by the ICAR test, the highest scores were observed in the group relying on the Integration strategy, $F(3, 341) = 5.607$, $p = .011$, $\eta^2 = 0.032$. However, significant differences for this measure were only found between the Integration and Not Sure strategies ($p = .034$).

Summary

In Study 3, we successfully replicated the main findings from Study 2, albeit with a different set of 120 unique choice problems. Consistent with our previous findings, participants who made less EV-consistent choices earned higher rewards. In addition, individuals with higher numeracy collected more rewards and made more choices within a limited timeframe, although these effects were not particularly strong. Notably, these effects were specific to numeracy and not to fluid intelligence. Moreover, the familiarity of the BNT did not influence the observed pattern of results.

Comparing two groups of participants—those with higher and lower numeracy—we observed that more numerate individuals sampled information more rapidly and dynamically. This approach allowed them to save time for solving additional choice problems. Interestingly, this pattern of exploration did not directly translate into changes in choice strategy, such as alterations in selecting gambles according to the predictions of the CPT or PH models. Instead, these participants adaptively relied on varying search strategies, initially focusing on reducing uncertainty and later on discovering unobserved outcomes.

Furthermore, individuals who demonstrated a metacognitive understanding of the task structure and who subjectively rated their performance as better, indeed received higher rewards. A qualitative analysis of the strategies described indicated that participants recognizing the effectiveness of the Random Selection strategy in this specific task environment earned more rewards. In addition, the average numeracy level was highest among those who adopted this random strategy.

Taken together, these findings suggest that people adaptively change their decision-making strategies (they apply dynamic sampling as well as uncertainty- and discovery-driven search strategies), often making recurring suboptimal choices that lead to superior long-term performance in the decision task. Skilled decision makers were able to discern and comprehend the structure of the task and decision environment, as evidenced by their metacognitive reports and adaptive exploration of choice problems.

Discussion

In our research, we found that decision makers who consistently made suboptimal random choices under time constraints accrued higher total rewards compared to those who opted for deliberate, energy-intensive normative decisions. While in an unconstrained environment, normative choices typically yield significantly higher rewards than repeated suboptimal decisions, our study revealed that individuals who recognized the efficacy of a rapid, random choice strategy in a resource-constrained environment gained more rewards. This was achieved through making recurring suboptimal choices, as opposed to adhering to a slower, more energy-intensive normative strategy.

We further examined whether skilled decision makers (i.e., individuals with higher numeracy) were able to discern the statistical structure of the time-constrained task environment (Dawes, 1979) and consequently adopt a strategy of recurring irrationality to achieve higher overall rewards compared to their less skilled counterparts. Our findings indicate that individuals with higher numeracy adaptively modified their exploration strategies in response to the alterations in the task environment. Specifically, they engaged in more dynamic and flexible information sampling and employed smart search strategies, predominantly focusing on reducing uncertainty and uncovering unobserved outcomes. This adaptability facilitated the development of a metacognitive understanding of both the structure of the decision task and the choice environment. As a result, they increasingly opted for random choices, which in turn led to higher total rewards.

These results are intriguing. Rational expectations based on probability rules, rational decision theory, and even common sense suggest that optimal decision making results from selecting an option for which the sum of the outcomes weighted by the probability of their occurrence is the highest (Baron, 2008). In contrast, we found the opposite effect—individuals who made more suboptimal choices objectively outperformed those more likely to make choices that maximize EV. In light of contemporary psychological models, such behavior cannot be regarded as rational (Stanovich, 1999). However, philosophical considerations of metatheories may shed light on this issue and prompt a rethink of the criteria of good decisions (Hammond, 2000). For example, the decision-making process may be coherent with standards of rationality (e.g., people make consistent choices maximizing EV), decisions may correspond to facts (e.g., people select alternatives that appear to be the most beneficial based on experienced outcomes), and decisions may be pragmatic (e.g., they increase the likelihood of achieving personal goals).

Let us consider the famous phenomenon of probability matching. In a typical *T*-maze experiment, a single mouse is placed at the starting point, and the reward (food) is randomly placed on the left side 80% of the time or on the right side 20% of the time. The optimal strategy for a mouse would be to turn left repeatedly. However, repeated observation throughout the years has documented that the mouse turns left approximately 80% of the time to match the presentation probability. This behavior is deemed as not optimal because the chances of finding food are $(0.80 \times 0.80 + 0.20 \times 0.20) = 68\%$, which is less than the optimal chance of 80% by using the rational strategy. Although looking at the choice behavior in light of the ecology surrounding the mouse, the strategy would not appear to be irrational. Assume that every mouse followed the rational strategy and repeatedly went to the left side, then the left side would be drastically more overcrowded, reducing the chance of getting food for everyone. Also, none of the mice would have exploited the less plentiful (but nevertheless available) food resources elsewhere. Therefore, the ecological perspective illustrates how following a strategy that creates overcrowding and at the same time is wasteful can easily be considered irrational (Mousavi & Kheirandish, 2014).

Similar examples can be found in the field of poker or chess, where players would deliberately deviate from the best option to surprise their competitors. In isolation, their choice might be regarded as suboptimal; however, in the context in which they had made that choice, the criterion of optimality fails to capture the rationale behind those choices. The modern-day term "rationality" comes from the Latin "ratio," meaning reason (Page, 2022). Colloquially, a rational decision is a decision that is reasonable or justifiable. However, the foundation of rationality is built on the back of efficient mathematics supported by principles such as completeness and transitivity (Baron, 2004; Schoemaker, 1982; Von Neumann & Morgenstern, 1944). Such a simple method allows researchers to theorize and evaluate the quality of individual choices, but sometimes, this approach prevents the researchers from thinking about the more extensive features or underlying reasons behind those choices (Lazear, 2000). Our results show why considering rationality in light of strict principles may not always capture the quality of overall choices. Importantly, we highlighted that choices could violate normative principles, yet they can be reasonable. This approach also corroborates the relationship between the psychological plausibility of individuals' behavior and its ecological effectiveness of it. Unlike the studies in the field of heuristics and biases (Tversky & Kahneman, 1974), ecological rationality (Gigerenzer & Selten, 2002), or the accuracy-effort trade-off framework (Payne et al., 1993), we demonstrate that making suboptimal choices not only approximates the standard normative choices but can also outperform them. We have found that decision makers who followed a normatively superior strategy earned significantly less overall rewards than decision makers who made more random choices under time constraints due to the time associated with following computationally complex processes.

Superior or optimal decision making cannot be fully realized without considering the constraints associated with task characteristics (e.g., time limits, uncertainty, opportunity cost) and human reasoning (e.g., resource limitation, cognitive thresholds; Bhui et al., 2021; Lieder & Griffiths, 2020). Our results indicate that skilled decision makers (i.e., more numerate individuals) were able to discern the statistical structure of the task environment and were able to take advantage of the constraints related to cognitive resources, temporal dynamics, and task complexity due to their superior numeric competency. In Study 1 (i.e., in unconstrained task structure), more numerate individuals spent more time on each problem, switched less frequently between options, and sampled more from each option to get a more accurate representation of the problem in order to find the normatively superior options than less numerate individuals. However, in a time-constrained environment, more numerate individuals significantly changed their decision strategy so that the metacognitive decision process and choice strategy reflected the changes that occurred in the decision environment.

In Studies 2 and 3, where participants were informed about the time constraints, individuals with higher numeracy demonstrated a more ecologically viable metacognitive decision process. This was characterized by more dynamic sampling (i.e., a greater number of samples and switches between gambles within a shorter time frame) and the adaptive reliance on search strategies, compared to their less numerate counterparts. Such processes further facilitated the decision-making strategy of more numerate individuals, enabling them to achieve higher overall rewards. In essence, skilled decision makers were more adept at discerning the statistical structure of the environment. Coupled with their superior numerical ability, they effectively leveraged the dynamics of the task to secure significantly greater rewards than less skilled decision makers. This effect can be explained by theoretical models positing that skilled decision making is driven by representative understanding (Cokely et al., 2018) and gist representation (Reyna & Brust-Renck, 2020) rather than rational optimization. Future research should delve into understanding precisely how metacognitive understanding of choice problems develops. Several candidate factors warrant detailed examination, including the impact of decision feedback, feedback on sampled outcomes, and the role of error monitoring in shaping this understanding (Yeung & Summerfield, 2012).

Since the studies of the current project employed only a limited set of choice problems in the gain domain (due to ethical issues), our findings cannot be directly generalized to the loss domain or other nonlottery-based tasks. Monetary lotteries are to decision sciences what fruit flies are to biology (Bateman et al., 2007), that is, they allow the foundations of decision making to be tested in controlled laboratory settings. We believe that our chosen approach provides a promising starting point to test whether recurring suboptimal choices lead to superior decision making in real-life problems or in scenarios involving losses where skilled decision makers should be less likely to make inferior choices. We can speculate that skilled decision makers in the loss or mixed domains in a time-constrained environment should make slower decisions, ultimately preventing them from making any choice at all. Additionally, it would be worthwhile to examine whether the effect we demonstrated persists in value-based decisions or in a within-subjects design. Such a design would enable more precise testing of the dynamics of strategy changes across different choice environments. Last, we controlled for fluid intelligence using progressive matrices to observe the effect of recurring irrationality among individuals with varying levels of numeracy. However, recent studies have shown that crystallized intelligence significantly influences how decision makers apply prior learning and past experiences in their decision-making process (Horn & Cattell, 1966; Shoots-Reinhard et al., 2021). This represents a limitation of our study. Future research should consider incorporating both fluid and crystallized intelligence into their designs.

Our study design raises further questions about the role of risk preferences in the decision-making task we employed. Specifically, it opens the possibility that more suboptimal choices might stem from increased risk-seeking behavior rather than an adaptive use of decision and search strategies. For instance, risk-seeking participants might simply take more risks by selecting a gamble without any prior information on outcomes. In our series of studies, we confined the choice problems to the gain domain, which minimizes the likelihood that risk preferences significantly influenced the search process and decisions. However, an analysis of qualitative data on metacognitive strategies revealed that about 4% of responses mentioned risk. These responses indicated a dynamic decision-making approach, where participants weighed the benefits of thorough analysis against the merits of quicker, less-informed decisions, particularly under time constraints and the lure of potential gains. They demonstrated a willingness to embrace risk when the anticipated payoff appeared to be substantial, adjusting their strategies in response to the evolving dynamics of the game and the repercussions of their choices. To conclusively determine whether risk-seeking behavior is a pivotal factor in such tasks, future research should explore choice problems in loss or mixed domains. This approach is essential due to the inherent challenges in inferring risk preferences from decisions from description (Wulff et al., 2015). The variability in sample sizes and the randomness of sample composition in decision-making tasks often result in each individual facing unique decision problems. This variety complicates the task of deducing risk preferences from their choices, as the diverse sets of information and options encountered by different individuals make it difficult to derive consistent conclusions about their risk-taking tendencies based on their decisions alone. In addition, a promising direction for future research involves systematically investigating how variations in EVs influence adaptive strategy selection and the effect we observed in the current research. Since previous research (Mondal & Traczyk, 2023a; Traczyk, Sobkow, et al., 2018) documented that the difference or ratio of EVs may impact the transition in decision-making strategies, it would be interesting to investigate whether skilled decision makers are capable of recognizing these distinctions and utilizing them to enhance their decision-making processes.

Finally, we believe that our findings will prompt scholars interested in decision making to rethink the criteria of good decisions as well as provide the impetus for redesigning boosting or nudging interventions aimed at supporting good choices (Hertwig & Grüne-Yanoff, 2017). Individuals and policymakers should be aware that, at least under some conditions, recurring suboptimal choices can result in superior decision making. Efforts to train individuals in the rational use of available cognitive resources, particularly when making decisions under environmental resource constraints, could enhance decision-making abilities in complex and dynamic settings. This could be especially beneficial for individuals with lower numerical abilities.

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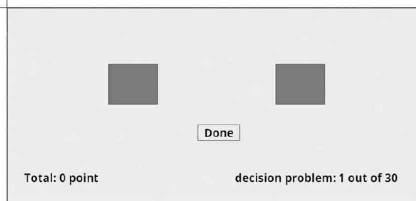
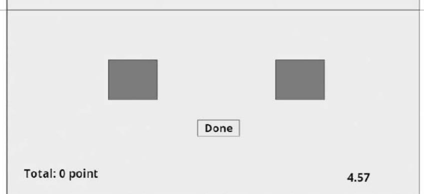
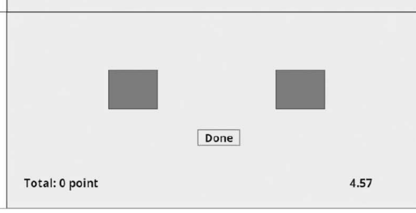
(*Appendix follows*)

Appendix

Experimental Design

Figure A1
Procedural Differences Between the Three Studies

Study	Screenshot	Information about the exact number of choice problems	Information about the elapsed time during the task	Choice Problems	Measures
Study 1		Yes	No	10 unique problems	Choice problems & BNT
Study 2		No	Yes	10 unique problems	Choice problems & BNT
Study 3		No	Yes	120 unique problems	Choice problems, BNT & ICAR

Note. BNT = Berlin Numeracy Test; ICAR = International Cognitive Ability Resource.

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10.3.1 Supplementary Materials

Recurring Suboptimal Choices Result in Superior Decision Making

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Supplementary online materials

Auxiliary study

Description

The purpose of this study was to investigate whether simply informing participants about a time limit, without additional cues like a timer, would be sufficient to prompt them to make faster, potentially suboptimal choices. To assess this, we asked participants to engage in the DfE task. During the instructions, they were told that the task had a time limit of 5 minutes, but no further details or time-related cues were provided.

In the auxiliary study, we observed a reversal of the results compared to Study 1, aligning more closely with the findings of Study 2. That is, the greater total reward was negatively associated with EV consistency (r = -.230, $p < .001$), the number of samples (r = -.641, $p < .001$), and Response time (r = -.474, $p < .001$), but positively with switching ratio (r = .398, $p < .001$) and the number of all decisions made (r = .939, $p < .001$). We found that numeracy was related to EV consistency (r = .153, p = .004) and switching ratio (r = -.107, p = .048). There was no evidence that more numerate people earned more money in Study 2.

In regression analysis, EV consistency predicted total gain in the opposite direction to this in Study 1 (b = -79.47, p = .025). In other words, participants who were less likely to make choices maximizing EV earned more money on the decision task. Additionally, people who put less effort into processing choice problems in the exploration stage drew fewer

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Table S1

Descriptive statistics and Pearson's r coefficients in the auxiliary study

Measure	Median	Mean	SD		1.	2.	3.	4.	5.	6.	7.
1. Total gain	132	157.36	107.66	r	-						
				p	-						
2. EV consistency	0.67	0.67	0.13	r	-0.23	-					
				p	<.001	-					
3. Number of samples	10.61	12.72	10.58	r	-0.641	0.218	-				
				p	<.001	<.001	-				
4. Switching ratio	0.25	0.35	0.27	r	0.398	-0.166	-0.457	-			
				p	<.001	0.002	<.001	-			
5. Response time	8.39	10.67	11.57	r	-0.886	0.33	0.836	-0.449	-		
				p	<.001	<.001	<.001	<.001	-		
6. Numeracy	1	1.25	1.24	r	-0.007	0.153	0.031	-0.107	0.03	-	
				p	0.899	0.004	0.562	0.048	0.582	-	
7. Number of decisions	28	36.17	24.41	r	0.939	-0.35	-0.672	0.421	-0.938	-0.012	-
				p	<.001	<.001	<.001	<.001	<.001	0.832	-

samples (b = -5.23, $p < .001$) in the DfE task and switched more frequently between payoff distributions (b = 57.13, p = .004), ended the study, on average, with a higher reward.

Table S2

Results of linear regression models predicting total gain in the auxiliary study.

	$R^2 = .43$					
Coefficient	b	SE	t	p	95% CI	
					LL	UL
Intercept	157.71	4.46	35.33	<.001	148.93	166.49
EV consistency	-79.47	35.32	-2.25	0.025	-148.95	-10
Numeracy	3.67	3.66	1	0.317	-3.54	10.88
Number of samples	-5.23	0.73	-7.16	<.001	-6.66	-3.79
Switching ratio	57.13	19.43	2.94	0.004	18.92	95.34
Response time	-0.5	0.56	-0.9	0.371	-1.6	0.6
EV consistency * Numeracy	-13.87	27.76	-0.5	0.618	-68.47	40.73

Summary

In summary, the mere mention of time constraints was sufficient to influence participants to make more suboptimal choices, leading to higher overall rewards. However, under these conditions, numeracy did not emerge as a significant factor. Contrary to expectations, more numerate individuals did not effectively balance decision accuracy against the number of decisions made to achieve greater rewards.

Decisions-from-Experience task (DfE) Structure

Figure S1

Schematic illustration of the general experimental procedure. A decision maker explores outcomes and their probabilities in a choice problem where they may earn 2 points with a probability of 45% or 12 points with a probability of 24%. The first sample from the gamble located on the left side of the screen has revealed a potential outcome of 2 points. The second sample has been drawn from the other gamble, and the outcome is 0 points. Next, the decision maker has learned that sampling the first gamble could earn 2, 2, and 0 points. After a total of 10 samples, the decision maker is ready to make a choice. They select a gamble on the left, resulting in a gain of 2 points. This feedback is followed by the next choice problem.

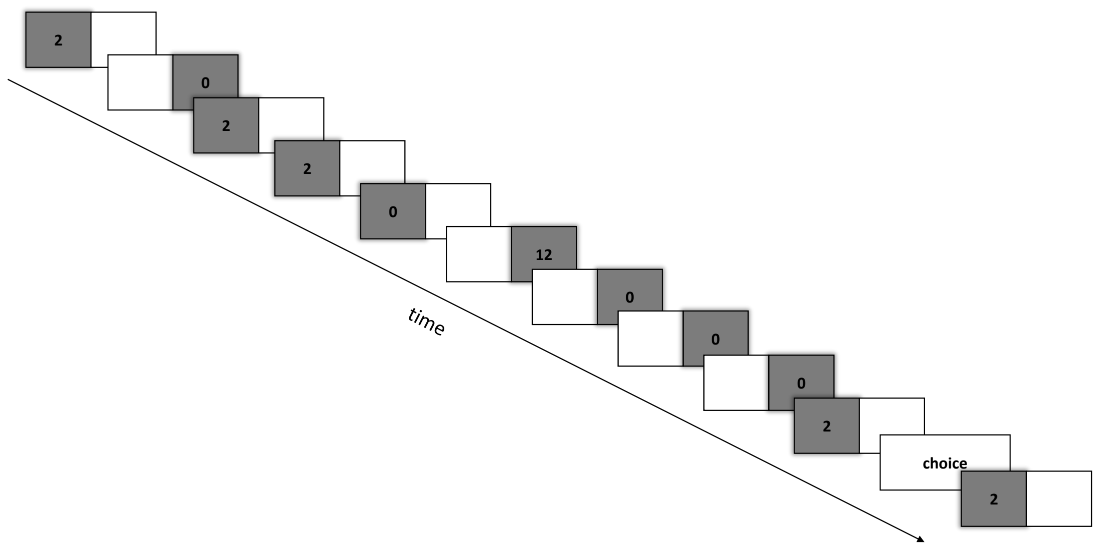

Figure S2

The mean number of choices consistent with the PH (priority heuristic) strategy is plotted as a function of the number of choice problems solved and numeracy. The shaded regions denote the 95% CI derived from a binomial distribution for consecutive binary choice problems. The point estimates (depicted as lines) within the 95% CI suggest that the decision strategy is statistically indistinguishable from a random selection strategy.

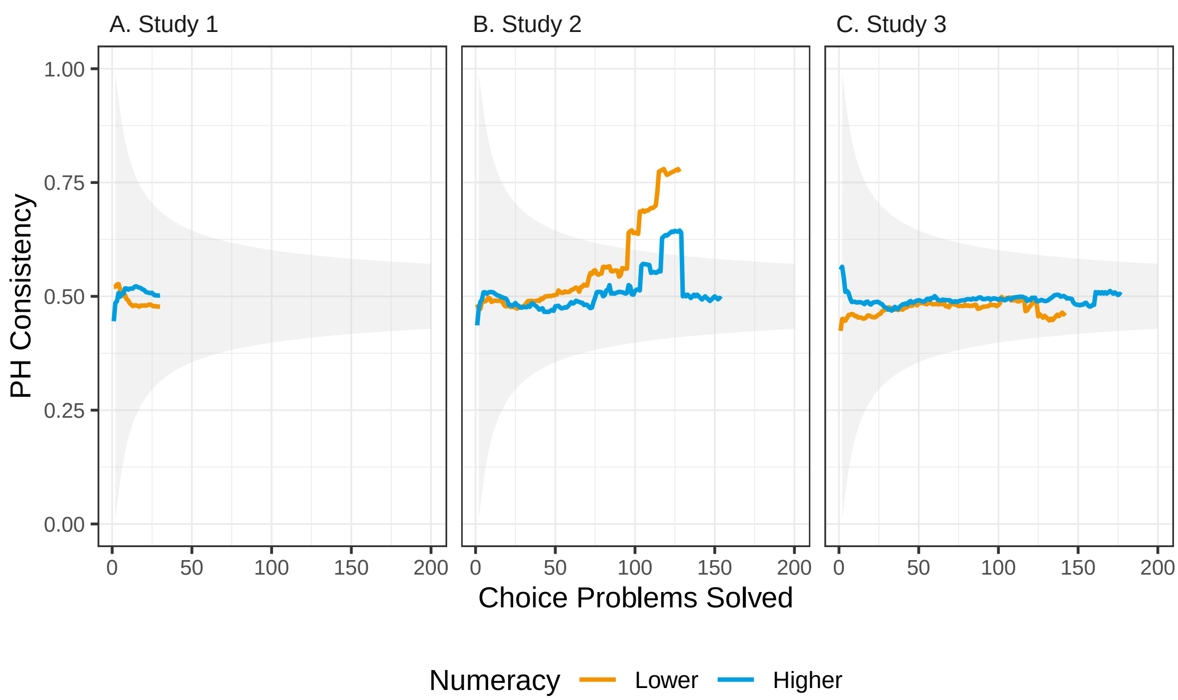

Table S3

Descriptive statistics in Study 1.

	Median	Mean	Std. Deviation
Total gain	135	135.757	31.812
EV consistency	0.7	0.7	0.124
Number of samples	13	13.863	8.203
Switching ratio	0.184	0.308	0.262
Response time	8.885	10.307	7.744
Numeracy	1	1.36	1.235
Number of decision	30	30	0

Table S4

Pearson's correlation coefficients among variables measured in Study 1.

Variable		1.	2.	3.	4.	5.	6.	7.
1. Total Gain	Pearson's r	-						
	p-value	-						
2. EV consistency	Pearson's r	0.266	-					
	p-value	<.001	-					
3. Number of samples	Pearson's r	0.125	0.326	-				
	p-value	0.019	<.001	-				
4. Switching ratio	Pearson's r	-0.009	-0.121	-0.492	-			
	p-value	0.866	0.023	<.001	-			
5. Response time	Pearson's r	0.139	0.199	0.77	-0.335	-		
	p-value	0.009	<.001	<.001	<.001	-		
6. Numeracy	Pearson's r	0.014	0.176	0.249	-0.262	0.145	-	
	p-value	0.789	<.001	<.001	<.001	0.007	-	
7. Number of decision	Pearson's r	NaN	NaN	NaN	NaN	NaN	NaN	-
	p-value	NaN	NaN	NaN	NaN	NaN	NaN	-

Note. The variance in Number of decision is equal to 0.

Table S5

Choice problem statistics of Study 1

Problem Number	oA1	pA1	oB1	pB1	nsampleA	nsampleB	Total Switching	Switching ratio	Sampling ratio	Choice Proportion of option A	EVA	EVB
1	15	0.71	10	0.97	6.1	5.24	2.56	0.35	1.07	0.56	10.65	9.7
2	19	0.45	12	0.65	6.74	5.51	2.42	0.32	1.06	0.58	8.55	7.8
3	5	0.68	16	0.45	6.09	6.13	2.52	0.31	1.09	0.26	3.4	7.2
4	2	0.27	8	0.39	6.86	6.11	2.76	0.31	1.13	0.17	0.54	3.12
5	12	0.24	2	0.45	8.46	5.94	2.91	0.3	1.15	0.75	2.88	0.9
6	4	0.79	10	0.03	6.1	9.7	2.79	0.28	1.18	0.85	3.16	0.3
7	16	0.23	2	0.39	8.38	6.23	2.98	0.29	1.15	0.77	3.68	0.78
8	2	0.12	16	0.14	8.9	8.87	3.39	0.27	1.23	0.23	0.24	2.24
9	16	0.45	7	0.63	6.73	5.57	2.44	0.32	1.08	0.68	7.2	4.41
10	14	0.21	8	0.44	8.52	6.44	2.78	0.29	1.14	0.52	2.94	3.52

Table S6

Descriptive statistics in Study 2.

	Median	Mean	Std. Deviation
Total gain	157	174.733	105.957
EV consistency	0.661	0.652	0.121
Number of samples	8.019	9.576	8.827
Switching ratio	0.257	0.349	0.259
Response time	5.723	7.642	8.22
Numeracy	1	1.517	1.249
Number of decision	35	40.509	24.563

Table S7

Pearson's correlation coefficients among variables measured in Study 2.

Variable		1.	2.	3.	4.	5.	6.	7.
1. Total gain	Pearson's r	—						
	p-value	—						
2. EV consistency	Pearson's r	-0.33	-					
	p-value	<.001	-					
3. Number of samples	Pearson's r	-0.63	0.401	—				
	p-value	<.001	<.001	-				
4. Switching ratio	Pearson's r	0.358	-0.102	-0.401	-			
	p-value	<.001	0.059	<.001	—			
5. Response time	Pearson's r	-0.611	0.293	0.853	-0.372	-		
	p-value	<.001	<.001	<.001	<.001	-		
6. Numeracy	Pearson's r	0.096	0.016	-0.024	-0.128	-0.043	-	
	p-value	0.037	0.382	0.328	0.008	0.212	-	
7. Number of decision	Pearson's r	0.939	-0.448	-0.651	0.333	-0.627	0.092	-
	p-value	<.001	<.001	<.001	<.001	<.001	0.044	-

Table S8

Choice problem statistics of Study 2

Problem Number	oA1	pA1	oB1	pB1	nsampleA	nsampleB	Total Switching	Switching ratio	Sampling ratio	Choice Proportion of option A	EVA	EVB
1	15	0.71	10	0.97	2.8	2.35	1.37	0.43	0.77	0.55	10.65	9.7
2	19	0.45	12	0.65	2.88	2.43	1.38	0.4	0.77	0.56	8.55	7.8
3	5	0.68	16	0.45	2.94	2.86	1.46	0.4	0.8	0.37	3.4	7.2
4	2	0.27	8	0.39	3.37	2.95	1.72	0.39	0.84	0.28	0.54	3.12
5	12	0.24	2	0.45	4.03	2.94	1.81	0.4	0.83	0.62	2.88	0.9
6	4	0.79	10	0.03	3.06	3.3	1.62	0.39	0.85	0.73	3.16	0.3
7	16	0.23	2	0.39	3.53	2.91	1.74	0.38	0.85	0.66	3.68	0.78
8	2	0.12	16	0.14	3.85	3.93	1.96	0.37	0.88	0.39	0.24	2.24
9	16	0.45	7	0.63	3	2.46	1.5	0.41	0.78	0.59	7.2	4.41
10	14	0.21	8	0.44	3.57	2.88	1.74	0.4	0.85	0.49	2.94	3.52

Table S9

Descriptive statistics in Study 3

	Median	Mean	Std. Deviation
Total gain	242	304.128	210.454
EV Consistency	0.533	0.543	0.102
Number of samples	8.303	9.454	7.429
Switching ratio	0.238	0.335	0.255
Response time	6.272	7.195	6.17
Numeracy	1	1.504	1.274
Number of decisions	35	43.858	29.643
Fluid intelligence	2	1.945	1.232
Subjective performance	2	2.464	20.353

Table S10

Pearson's correlation coefficients among variables measured in Study 3

Variable		1.	2.	3.	4.	5.	6.	7.	8.	9.
1. Total gain	Pearson's r	-								
	p-value	-								
2. EV consistency	Pearson's r	-0.16	-							
	p-value	0.003	-							
3. Number of samples	Pearson's r	-0.688	0.199	-						
	p-value	<.001	<.001	-						
4. Switching ratio	Pearson's r	0.305	-0.065	-0.433	-					
	p-value	<.001	0.227	<.001	-					
5. Response time	Pearson's r	-0.676	0.263	0.859	-0.372	-				
	p-value	<.001	<.001	<.001	<.001	-				
6. Numeracy	Pearson's r	0.103	-0.073	-0.04	-0.157	-0.041	-			
	p-value	0.028	0.089	0.228	0.002	0.224	-			
7. Number of decisions	Pearson's r	0.963	-0.194	-0.719	0.306	-0.709	0.101	-		
	p-value	<.001	<.001	<.001	<.001	<.001	0.030	-		
8. Fluid intelligence	Pearson's r	-0.029	-0.012	0.021	-0.131	-0.017	0.307	-0.058	-	
	p-value	0.59	0.82	0.691	0.015	0.749	<.001	0.284	-	
9. Subjective performance	Pearson's r	0.464	-0.006	-0.268	0.102	-0.256	0.046	0.412	0.023	-
	p-value	<.001	0.907	<.001	0.059	<.001	0.394	<.001	0.671	-

Table S11

Results of linear regression models predicting total gain in Study 3.

	Estimate	SE	t	p
(Intercept)	304.128	11.216	27.116	<.001
EV consistency	-326.705	110.282	-2.962	0.003
ICAR	-5.294	9.119	-0.581	0.562

Table S12

Results of linear regression models predicting total gain in Study 3.

	Estimate	SE	t	p
(Intercept)	304.370	8.037	37.869	<.001
EV consistency	27.669	82.053	0.337	0.736
Numeracy	15.662	6.752	2.320	0.021
Number of samples	-10.984	2.185	-5.028	<.001
Switching ratio	17.005	35.823	0.475	0.635
Response time	-11.488	2.593	-4.430	<.001
ICAR	-9.065	6.895	-1.315	0.190
EV consistency * Numeracy	25.775	63.740	0.404	0.686

Table S13

Choice problem statistics of Study 3

Problem Number	oA1	pA1	oB1	pB1	nsampleA	nsampleB	Total Switching	Switching ratio	Sampling ratio	Choice Proportion of option A	EVA	EVB
1	23	0.23	6	0.82	3.16	2.59	1.36	0.4	0.78	0.5	5.29	4.92
2	10	0.83	16	0.69	2.78	2.37	1.13	0.34	0.78	0.44	8.3	11.04
3	25	0.23	14	0.37	3.18	2.85	1.55	0.37	0.82	0.56	5.75	5.18
4	2	0.42	9	0.17	2.88	3.06	1.93	0.41	0.85	0.4	0.84	1.53
5	20	0.24	6	0.7	3.19	2.17	1.32	0.39	0.8	0.53	4.8	4.2
6	26	0.15	5	0.71	3.2	2.35	1.64	0.38	0.87	0.42	3.9	3.55
7	21	0.28	4	0.77	3.29	2.47	1.53	0.4	0.8	0.49	5.88	3.08
8	10	0.79	25	0.56	2.79	2.72	1.38	0.39	0.78	0.37	7.9	14
9	20	0.12	3	0.75	3.47	2.62	1.57	0.37	0.9	0.45	2.4	2.25
10	26	0.52	14	0.7	2.89	2.74	1.45	0.41	0.76	0.58	13.52	9.8
11	3	0.97	20	0.27	2.98	3.39	1.38	0.35	0.83	0.42	2.91	5.4
12	14	0.2	2	0.9	2.92	2.29	1.47	0.41	0.8	0.55	2.8	1.8
13	18	0.49	29	0.46	2.59	2.2	1.23	0.37	0.75	0.44	8.82	13.34
14	9	0.73	24	0.36	2.54	2.29	1.73	0.49	0.74	0.48	6.57	8.64
15	18	0.55	14	0.67	2.55	2.21	1.06	0.36	0.71	0.62	9.9	9.38
16	14	0.34	8	0.47	3.21	2.72	1.75	0.36	0.81	0.61	4.76	3.76
17	11	0.63	29	0.26	3.27	2.98	1.64	0.4	0.83	0.56	6.93	7.54
18	8	0.61	6	0.73	2.97	3.45	1.77	0.42	0.82	0.63	4.88	4.38
19	10	0.57	24	0.33	3.1	3	1.28	0.35	0.76	0.5	5.7	7.92
20	6	0.86	17	0.46	2.67	2.48	1.3	0.41	0.72	0.39	5.16	7.82
21	10	0.4	27	0.19	3.33	2.99	1.36	0.37	0.78	0.48	4	5.13
22	2	0.41	5	0.21	3.01	3.12	1.46	0.36	0.8	0.52	0.82	1.05
23	9	0.8	21	0.45	2.83	2.78	1.29	0.39	0.79	0.47	7.2	9.45
24	26	0.11	7	0.31	3.14	2.38	1.51	0.43	0.77	0.48	2.86	2.17
25	19	0.71	9	0.98	3.03	2.26	1.05	0.36	0.71	0.62	13.49	8.82
26	21	0.45	11	0.56	2.81	2.38	1.5	0.46	0.7	0.5	9.45	6.16
27	7	0.72	14	0.48	2.46	2.29	1.37	0.43	0.71	0.37	5.04	6.72
28	9	0.76	19	0.53	2.9	2.62	1.94	0.47	0.87	0.43	6.84	10.07
29	4	0.44	28	0.11	2.84	3.18	1.82	0.38	0.82	0.55	1.76	3.08
30	6	0.47	2	0.86	2.98	2.5	1.44	0.4	0.83	0.54	2.82	1.72
31	17	0.24	4	0.71	3.35	2.6	1.59	0.38	0.7	0.55	4.08	2.84
32	8	0.83	30	0.3	2.58	2.75	1.3	0.41	0.72	0.5	6.64	9
33	20	0.82	15	0.94	2.3	2.05	1.17	0.41	0.77	0.61	16.4	14.1
34	29	0.03	1	0.58	3.47	2.09	1.35	0.37	0.79	0.44	0.87	0.58
35	16	0.67	28	0.46	3.39	3.27	1.56	0.37	0.89	0.41	10.72	12.88
36	5	0.13	2	0.27	4.2	3.65	1.61	0.33	0.92	0.59	0.65	0.54
37	18	0.82	27	0.66	2.54	2.64	1	0.37	0.79	0.45	14.76	17.82
38	16	0.49	7	0.69	2.59	1.97	1.32	0.4	0.77	0.68	7.84	4.83
39	15	0.9	24	0.76	2.23	2.26	1.34	0.41	0.79	0.32	13.5	18.24
40	27	0.86	30	0.81	2.89	2.43	1.19	0.37	0.78	0.41	23.22	24.3
41	15	0.33	5	0.74	3.18	2.7	1.42	0.34	0.88	0.56	4.95	3.7
42	4	0.67	23	0.19	2.38	3.99	1.31	0.39	0.71	0.53	2.68	4.37
43	10	0.92	22	0.54	2.88	2.77	1.22	0.39	0.77	0.36	9.2	11.88
44	12	0.71	29	0.55	2.2	1.97	1.19	0.4	0.72	0.44	8.52	15.95
45	27	0.36	14	0.56	3.47	2.65	1.34	0.41	0.79	0.52	9.72	7.84
46	24	0.73	19	0.85	2.83	2.26	1.38	0.4	0.76	0.61	17.52	16.15
47	5	0.16	1	0.42	3.21	2.58	1.74	0.39	0.76	0.57	0.8	0.42
48	7	0.62	24	0.32	2.72	2.53	1.3	0.39	0.75	0.52	4.34	7.68
49	18	0.13	4	0.45	3.7	2.88	1.75	0.37	0.83	0.48	2.34	1.8
50	20	0.32	13	0.47	2.97	2.56	1.54	0.42	0.76	0.52	6.4	6.11
51	20	0.83	27	0.63	2.59	2.54	1.18	0.37	0.76	0.46	16.6	17.01
52	28	0.4	9	0.68	2.85	2.41	1.39	0.4	0.78	0.59	11.2	6.12
53	21	0.21	9	0.34	2.92	2.66	1.3	0.39	0.83	0.53	4.41	3.06
54	16	0.67	30	0.37	2.61	2.5	1.29	0.36	0.78	0.54	10.72	11.1
55	3	0.62	7	0.33	2.89	2.77	1.16	0.32	0.85	0.46	1.86	2.31
56	15	0.72	23	0.59	2.22	2.08	1.15	0.42	0.71	0.43	10.8	13.57
57	28	0.49	10	0.75	2.48	2.16	1.45	0.44	0.77	0.6	13.72	7.5
58	22	0.13	5	0.42	3.1	2.57	1.67	0.42	0.8	0.53	2.86	2.1
59	2	0.32	25	0.04	2.93	2.98	1.83	0.44	0.82	0.57	0.64	1
60	12	0.93	25	0.8	2.71	2.65	1.43	0.4	0.84	0.29	11.16	20
61	24	0.04	27	0.02	3.61	3.58	1.72	0.35	0.82	0.63	0.96	0.54
62	18	0.08	22	0.04	4.21	4.33	2.47	0.38	0.95	0.59	1.44	0.88
63	26	0.28	30	0.18	3.3	2.97	1.39	0.37	0.83	0.56	7.28	5.4
64	17	0.15	16	0.19	2.82	2.58	1.31	0.42	0.79	0.54	2.55	3.04
65	11	0.11	10	0.16	3.06	2.8	1.35	0.35	0.73	0.54	1.21	1.6
66	29	0.15	24	0.24	3.49	3.29	1.98	0.37	0.83	0.58	4.35	5.76
67	21	0.53	22	0.45	3.29	2.93	1.28	0.34	0.74	0.53	11.13	9.9
68	27	0.42	28	0.39	2.92	2.67	1.05	0.32	0.7	0.5	11.34	10.92
69	2	0.06	3	0.03	3.82	3.98	2.2	0.39	0.82	0.59	0.12	0.09
70	22	0.2	25	0.1	3.54	3.3	1.57	0.32	0.94	0.66	4.4	2.5
71	28	0.31	19	0.75	2.75	2.2	1.56	0.44	0.79	0.44	8.68	14.25
72	27	0.17	29	0.09	2.86	2.9	1.72	0.4	0.82	0.58	4.59	2.61
73	24	0.22	21	0.29	3.26	2.9	1.79	0.45	0.87	0.54	5.28	6.09
74	25	0.36	28	0.29	3.47	3.47	1.42	0.33	0.89	0.53	9	8.12
75	3	0.1	4	0.04	3.95	3.87	2.03	0.36	0.88	0.59	0.3	0.16
76	13	0.7	12	0.95	3.13	2.82	1.48	0.4	0.81	0.48	9.1	11.4
77	10	0.22	9	0.29	3.38	2.93	1.88	0.4	0.84	0.57	2.2	2.61
78	19	0.4	18	0.5	3.33	2.88	1.73	0.39	0.83	0.59	7.6	9

79	23	0.48	21	0.56	3.05	2.73	1.25	0.35	0.8	0.58	11.04	11.76
80	19	0.64	17	0.87	2.59	2.14	1.13	0.41	0.72	0.57	12.16	14.79
81	21	0.27	20	0.33	2.91	2.74	1.43	0.4	0.81	0.58	5.67	6.6
82	23	0.62	24	0.55	3.01	2.39	1.4	0.39	0.7	0.47	14.26	13.2
83	23	0.65	22	0.72	2.7	2.46	1.4	0.4	0.8	0.52	14.95	15.84
84	25	0.91	27	0.83	3.41	3.05	1.27	0.39	0.78	0.46	22.75	22.41
85	27	0.12	22	0.19	3.23	3.07	1.82	0.4	0.83	0.49	3.24	4.18
86	23	0.65	24	0.6	3.15	3.05	1.35	0.38	0.85	0.55	14.95	14.4
87	21	0.19	27	0.1	3.46	3.12	1.66	0.36	0.8	0.6	3.99	2.7
88	14	0.26	16	0.16	2.74	2.71	1.18	0.32	0.69	0.55	3.64	2.56
89	24	0.19	21	0.27	2.88	2.51	1.62	0.42	0.8	0.47	4.56	5.67
90	17	0.3	15	0.38	3.35	3.04	1.91	0.37	0.83	0.58	5.1	5.7
91	29	0.14	27	0.19	4.32	3.81	1.79	0.31	0.83	0.55	4.06	5.13
92	29	0.16	23	0.3	2.85	2.36	1.34	0.36	0.78	0.4	4.64	6.9
93	15	0.72	14	0.82	3.05	2.6	1.48	0.4	0.78	0.6	10.8	11.48
94	25	0.36	19	0.79	3.36	2.21	1.21	0.37	0.76	0.41	9	15.01
95	21	0.32	22	0.22	3.16	3.05	1.5	0.38	0.78	0.54	6.72	4.84
96	11	0.17	14	0.08	3.93	3.59	1.88	0.36	0.85	0.64	1.87	1.12
97	9	0.09	11	0.05	2.92	3.24	1.83	0.43	0.76	0.65	0.81	0.55
98	9	0.49	10	0.39	3.5	3.25	1.13	0.31	0.79	0.53	4.41	3.9
99	24	0.07	19	0.15	2.69	2.3	1.69	0.42	0.68	0.58	1.68	2.85
100	19	0.09	15	0.17	3.49	3.06	1.92	0.37	0.82	0.49	1.71	2.55
101	25	0.35	9	0.99	2.76	2.42	1.49	0.41	0.79	0.55	8.75	8.91
102	26	0.16	27	0.09	3.45	3.18	1.7	0.38	0.8	0.56	4.16	2.43
103	26	0.49	27	0.44	2.97	2.42	1.17	0.36	0.74	0.42	12.74	11.88
104	25	0.23	27	0.15	3.19	3.04	2.01	0.4	0.78	0.63	5.75	4.05
105	17	0.08	16	0.11	3.04	2.67	1.56	0.39	0.78	0.56	1.36	1.76
106	30	0.03	25	0.07	4.17	3.59	1.51	0.31	0.85	0.5	0.9	1.75
107	15	0.2	14	0.23	3	2.82	1.63	0.37	0.84	0.55	3	3.22
108	10	0.34	9	0.43	3.63	2.88	1.65	0.4	0.78	0.67	3.4	3.87
109	22	0.44	25	0.35	2.7	2.16	1.21	0.38	0.77	0.53	9.68	8.75
110	19	0.58	17	0.67	2.9	2.58	1.48	0.41	0.85	0.52	11.02	11.39
111	24	0.05	23	0.08	2.83	2.41	1.41	0.4	0.79	0.5	1.2	1.84
112	18	0.17	19	0.11	3.19	3.02	1.6	0.43	0.8	0.56	3.06	2.09
113	17	0.33	19	0.25	2.89	2.67	1.23	0.35	0.76	0.57	5.61	4.75
114	28	0.03	19	0.07	3.87	3.26	2.4	0.38	0.86	0.41	0.84	1.33
115	23	0.45	21	0.66	2.68	2.31	1.26	0.37	0.73	0.46	10.35	13.86
116	10	0.02	7	0.05	3.34	3.14	2.29	0.43	0.8	0.58	0.2	0.35
117	29	0.7	30	0.64	3.07	3.06	1.37	0.36	0.79	0.49	20.3	19.2
118	29	0.61	28	0.65	2.72	2.5	1.51	0.43	0.65	0.49	17.69	18.2
119	5	0.05	8	0.02	3.9	3.63	1.86	0.34	0.87	0.58	0.25	0.16
120	28	0.72	27	0.77	3.07	2.78	1.38	0.33	0.77	0.57	20.16	20.79